Mathematical Modelling and Problem Solving Project

Introducing a New Skills Module into the Year 1 Curriculum

In the School of Physics and Astronomy, Leeds University

Project Leader: **Professor Mike Savage**

Foreword

This report concerns the introduction of a new and novel skills module into the first year of the undergraduate curriculum in the School of Physics and Astronomy at the University of Leeds, in which the primary aim is to develop students' ability to solve physical problems using mathematical modelling and mathematics.

The report has been prepared for the HE STEM Programme who have generously supported the project and funded an in-depth evaluation study by Professor J. Williams from the School of Education, University of Manchester.

Both the timing and relevance of the work done at Leeds, including its evaluation, has to be seen in a greater context where it forms one strand of a more extensive National HE STEM project that involves 13 Departments in 8 universities. Led by Professor Savage, all 13 partners are committed to finding ways of developing and enhancing students' mathematical modelling and problem solving skills.

05/12/2012.

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Mathematical Modelling and Problem Solving

A National HE STEM Project

(with 13 Partners across 8 Universities)

Project Aims and Rationale

This HE STEM project "Mathematical Modelling & Problem Solving" aims to equip first year STEM undergraduates with a high level, transferable skill – "the ability to solve problems in science and engineering by setting up mathematical models and using mathematics"

Such skills are highly valued by both employers and researchers:

"The ability to solve physical problems using mathematical modelling and mathematics are invaluable attributes for new undergraduates entering an engineering industry"

Dr Alan Stevens,
Rolls Royce
Maths Modelling Group

"A key attribute for engineering graduates is their ability to apply theoretical knowledge to a real problem to create a solution. Modelling skills are key to this process."

Professor Barry Clarke,
President (elect)
Institution of Civil Engineers

Thirteen Project Partners in 8 Universities

There are currently 13 HE STEM Departments (Physics, Mathematics and Engineering spread across 8 universities, Appendix[1], that fully recognise the importance of modelling and problem solving skills for STEM undergraduates and are now actively engaged in introducing such skills into their curriculum in ways that are most suitable for their needs. This approach has enabled the 13 departments to implement tailored approaches based on agreed concepts.

There are, in fact, several ways of introducing modelling that are particularly appropriate for engineering, applied mathematics and physics students, of which Newtonian mechanics is one which is used in the School of Physics and Astronomy at the University of Leeds.

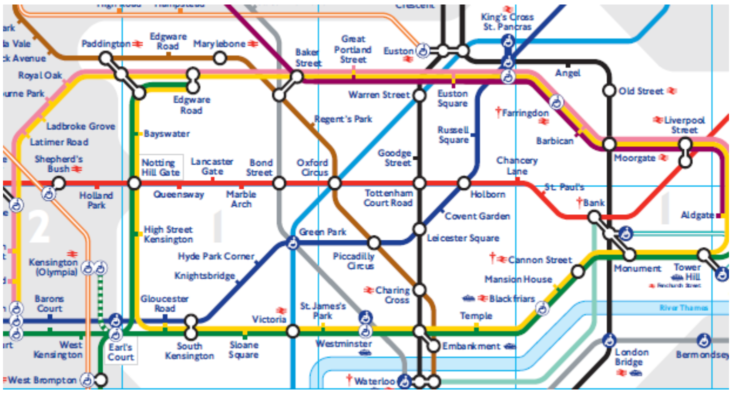
2. Models and Modelling

What is a model? ***What is Modelling?***

What has modelling got to do with problem solving?

2.1 The concept of a 'model'.

This London tube map generally comes in useful for visitors who need to get around the capital



The map is in fact a **model** in the sense of an approximation to reality; one which has the essentials that the traveller needs to know– tube lines, tube stations, main line stations...

So a model is an approximation or representation of reality which is useful for the task or problem in hand i.e. "how to plan a route?"

2.2 What is Modelling?

It is a process that begins once a problem has been formulated

In the following example; the aim is to move (pull) a heavy object across rough ground with the least effort!

One interesting question/problem would be : **What is the best pulling angle?**

The next step is to 'set up a model'!

2.3 Two Modelling Skills:

(i) Setting Up a Model

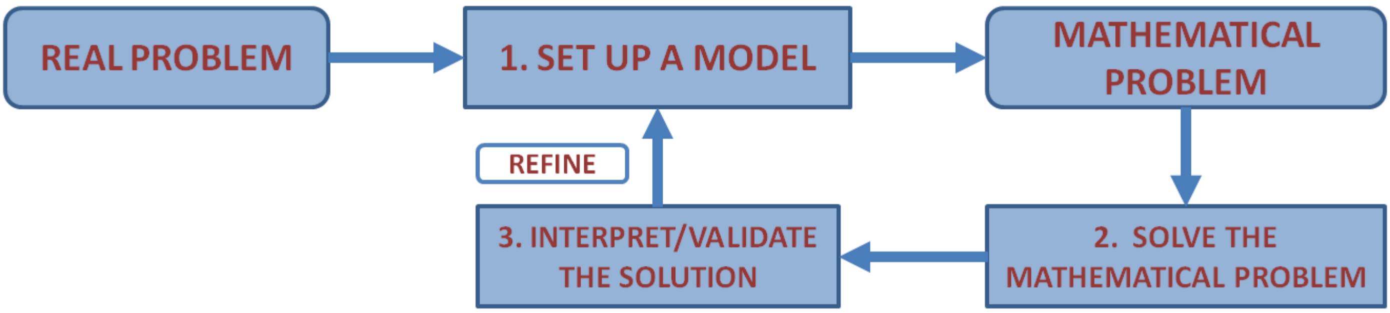
The idea of "setting up a model" is to convert a **real problem** into a **mathematical problem**

Setting up a model in Newtonian mechanics involves:

- Making assumptions
- Drawing a diagram
- Introducing variables
- Applying a law or principle

(ii) Multi-Stage Modelling

Starting with a real problem, multi-stage modelling involves the execution of a full modelling investigation by following the stages of a multi-stage **modelling cycle**



- **Setting up a Model"** is the <u>first stage</u> of the modelling cycle.
- **The <u>second stage</u>** is to solve the mathematical problem and
- **The <u>third</u>** is to interpret/validate the solution
- **Finally;** having gone around the cycle once, it is often necessary to seek an "improved solution" by **refining the model** and repeating the process again – and perhaps yet again!

3. Mathematical Modelling and Problem Solving: *Key features of the module*

A new module, "Mathematical Modelling and Problem Solving in Mechanics" was launched in September, 2011, at the University of Leeds for 62 first year physicists – all of whom had high grades in both A level physics and mathematics – yet had studied only 1 module of Newtonian mechanics (at most) in the sixth form.

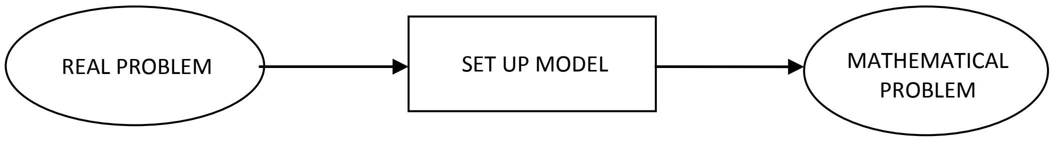
Aims and Objectives of new module, Mathematical Modelling and Problem Solving

- To introduce 62 students to various topics in Newtonian mechanics in order to provide them with the relevant background material that they had not covered in the sixth form (semester 1)
- To use Newtonian mechanics as the vehicle for introducing students to the two distinct modelling skills outlined in the General Introduction. These skills were continually practiced and developed during Semester 1 and Semester 2.

3.1 Semester 1.

At the start of Semester 1, students completed an evaluation questionnaire – designed by the evaluator, Julian Williams, for assessing and benchmarking their mathematical modelling skills. At this time, it was clear that the ability of students to tackle and solve problems was extremely limited since they had met few concepts and also had little experience of problem solving in mechanics in the sixth form.

During this first semester students were introduced to key concepts and problem solving techniques in Newtonian mechanics. In the workshops they worked in groups of 2, 3 or 4 on problems (on each of 4 worksheets) where the primary aim was to convert the 'real Problem' into a 'mathematical problem' by setting up a model.



Students were encouraged to interact with each other – talking, sharing, helping – and with tutors in order to "set up the model" and so produce a mathematical problem which was then solved using relevant mathematical methods and techniques.

In January, 2012, students sat a 1½ hour written examination - covering questions/problems, similar to those they had seen on the worksheets. The examination was designed to assess their ability to tackle physical problems by "Setting up Models" and "Finding Mathematical Solutions" – and in some cases "Interpreting Results"

3.2 Semester 2.

Students were initially introduced to the various stages of a "modelling cycle" (see Introduction) and the use of this was illustrated on a DVD of an investigation into circular motion at Alton Towers that features Carol Vorderman, see www.transmaths.org/mmps/. Ms Vorderman considers the 'waveswinger ride' (also referred to as the “chair-o-plane”) and addresses the problem of “Why do chairs swing out as speed increases?” – showing how the various stages of the modelling cycle are used to complete the investigation- beginning with a simple model and subsequently refining it. Students were then invited to select and then investigate one of the following modelling projects.

(i) investigate the motion of cars/bikes on horizontal tracks (banked/ unbanked),
(ii) investigate the motion of cars/bikes on tracks with vertical circles

Finally students submitted an individual report of their investigation that followed the stages of the modelling cycle and included an abstract.

<u>Conclusions regarding Modelling Projects/Investigations and Students Reports</u>

- Once engaged with their group, students soon appreciate the benefits of working together on a modelling project.
- A modelling project offers plenty of scope for each individual student and group to show just what they can do and achieve.
- Writing a report of an investigation is a valuable asset when going for a job interview – by demonstrating evidence of key skills that employers value!

3.3 Evaluation

Julian Williams and his evaluation team made 3 visits to Leeds during Semester 2 in order to

(i) observe/film students at work on their project
(ii) present students with a second modelling questionnaire – designed so as to reveal what progress students had made in understanding and developing their modelling skills
(iii) discuss modelling projects with individual students during filmed interviews; (accessible on www.transmaths.org/mmps/.)

An Evaluation Report from Professor Williams follows in Section 4.

4. Evaluation Report: Professor J. Williams

Mathematical modelling and problem solving: a first year module for Leeds Physics students

Introduction

I was asked to independently evaluate the development work in the School of Physics at the University of Leeds involving Professor Savage’s (and his colleagues’) teaching of “Mathematical modelling and problem solving” to first year students; the aim of the evaluation was to provide an independent perspective on this development, with a view to helping those involved to improve the course, and to help other pioneers that might follow in their footsteps.

We intended especially to gather together materials that would exemplify the students' work and learning outcomes. But also through discussion with the team at Leeds and elsewhere we intended to try to make value judgments about how the work might best be developed by those that follow.

It is important for a proper evaluation to understand the very particular aims of the work Prof Savage is trying to develop: the notion of mathematical modelling and problem solving (hereafter MM&PS) in the field of Newtonian mechanics has a particular form, one that many have argued is particularly relevant for the HE STEM sector, for it is argued it provides an arena in which students may pose and solve open-ended problems by modelling with Science (Newton's laws, and force laws) and Mathematics (geometry, equations) seriously for the first time in their intellectual development (see Crighton, 1975).

Before proceeding, I should also make transparent here my own history as a committed proponent of MM&PS along with Prof Savage and others: I want this to work. But also it is relevant to be aware that I have been involved in trying to make MM&PS 'work' in the past, and failed. A proper history of the rise and fall of "Mathematical applications and modelling" in England is yet to be written, but a brief remark is in order. Many know that (and some say they understand why) the pre-eminent position of British work in applied mathematics has been increasingly brought into question in the last 50 years. Not so many are aware that a somewhat similar trajectory has affected the field of "applied mathematics / modelling in education" since the 1990s. Looking at the publications of the ICTMA, the international organisation of educators researching and developing 'applied mathematics' and 'mathematical modelling' world-wide, one sees that the whole movement was led and dominated by English and British innovators for a decade or more, while Europeans (especially the Freudenthal trend) later reshaped and enhanced this movement. But looking now at the contributions to this field one sees that the far East is now beginning to dominate and English work has seriously declined if not almost disappeared, having lost its momentum in the 1990s (reflecting the disappearance of modelling from universities and senior schools curricula, including highly successful modelling courses run by MEI, and SMP A level programmes). As a final example: almost the entire first few conferences of ICTMA was dominated by English contributions and papers; in the last ICTMA proceedings one finds 2 out of 53 of the paper contributions from English authors (Wake, and Hoyles, ICTMA-13, 2010).

I therefore approach this evaluation as an enthusiast but also a realist. This independent evaluation will consequently be evidence-based and critical, but optimistic - as I share many of the values of the proponents of this innovation (for many reasons not entered into here, but see Williams, 2012).

Methodology and methods

My approach has been to evaluate the extent to which the relevant students were, and might be, brought to genuinely solve problems in Physics using mathematical modelling, and so became, or might be made to, become aware of modelling as a process, and therefore to align themselves with this modelling process. This notion of 'genuine' problem solving is complex and arguable, so the evaluation is necessarily nuanced in this regard.

The course was studied in a quasi-ethnographic manner, being informed by: the course documents, observations on visits to the course sessions on three occasions, examining the learning outcomes as evidenced in the students' coursework, and cross checking these observations with interviews with some (five) key students as well as Prof Savage.

In addition, my team and I developed two instruments to try to measure the learning outcomes of students as a result of the course in ways that would make some sense across courses in different disciplines, including mathematics, Science and Engineering. With the first Likert type questionnaire instrument we constructed a measure of "disposition towards mathematical modelling and problem solving" (an affective measure of how positive the students feel about MM&PS). The other measure resulted from the second evaluation instrument and was tentatively labelled "awareness of mathematical modelling and problem solving", i.e. a measure of how aware the students are regarding the process(es) that experts say are involved in MM&PS. Our usual validation methodology used in the past for developing such measures (see e.g. Pampaka, Williams, & Hutcheson, 2011; Pampaka, Williams, et al, 2012) established that the first of these was a reliable and valid measure, while the latter – although promising – needs more substantial development (see interim report for the HE-STEM project by Pampaka & Williams, 2012 on www.transmaths.org website).

Results

Objective measures

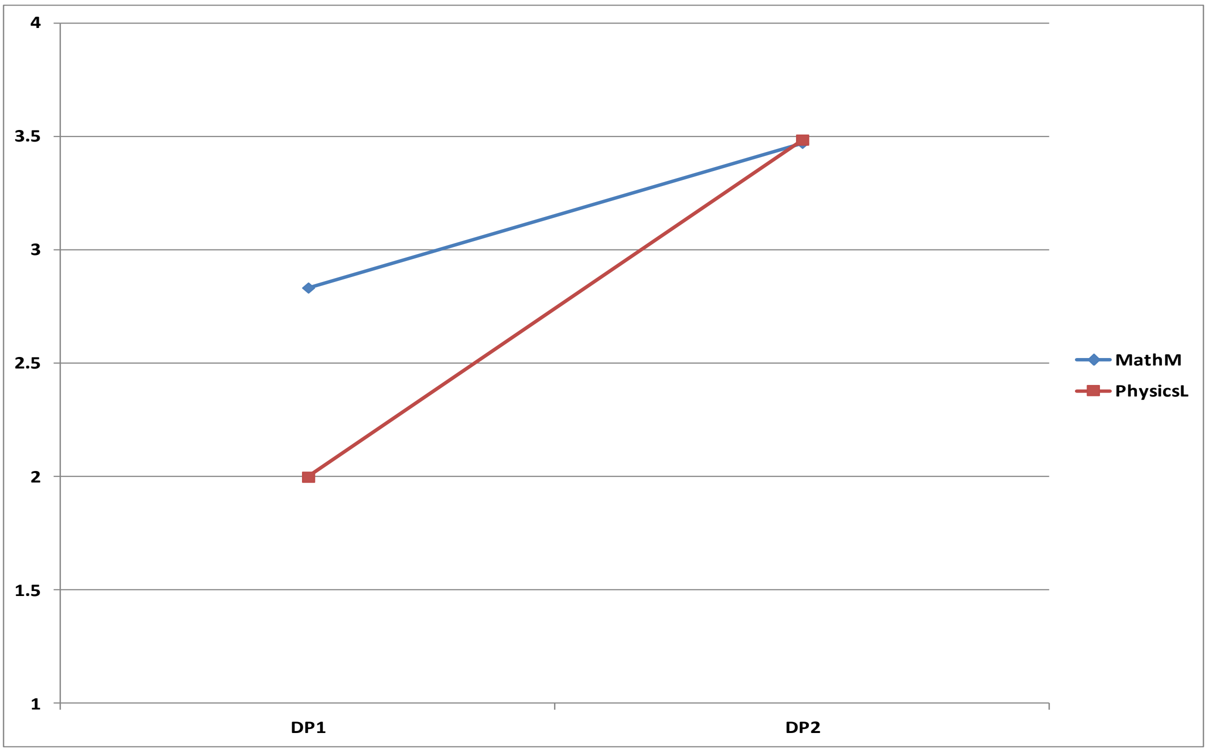


Figure 1: the pre-course and post-course disposition scores (in logits) for the group of Physics students compared to a group of mathematics students following MM&PS courses. For further r details see the report on the Transmaths MM&PS website at http://transmaths.org/mmps/MMPS Evaluation Report for MATHS MODELING-July2012.pdf

The data and findings can be seen in the interim report describing our construction and analysis of objective measures of ‘disposition towards’ and ‘awareness of’ MM&PS. A value-added model showed that the Physicists involved in this study did significantly increase their scores on disposition, as did a somewhat similar (i.e. first year, similarly prepared A-graded) cohort of undergraduate Mathematics students in another (but not too dissimilar) university institution. (This analysis is provided in the report on the website.) This statistically significant improvement compared well with others improvements, mainly as their initial pre-course scores were low compared to a comparative group of mathematicians (at the other university).

The findings arising from the awareness instrument were inconclusive: we found the questionnaire data inconsistent with interview data and concluded this measure would need more development. This is on-going work: in the 2012-13 evaluation it is planned to gather further data from the wider project participants to validate this instrument, with a view to reporting similar effects of instruction such as that shown in Figure one.

Case study in the field

The students’ response to the course (largely as reported by the staff teaching the course, but also based on our observations of the course teaching and interviews with 6 students) showed that it IS possible to teach first year Physics to a ‘modelling’ agenda, engaging students in groups posing and solving problems in ‘open’ contexts that are motivating - but that such a course makes a variety of demands on students that contrast with much else in their experience and courses that should not be underestimated:

- In general the students’ previous experience seems to have - and elsewhere in their course continues to reflect - a ‘content-led’ agenda, i.e. skills and modelling (if they exist) take second place. This was reflected in some students’ comments that there was insufficient ‘new material’ in the course. Thus, there is a requirement that students engage with learning with a new perspective on the intended outcomes, implying a critique of their former (quite successful) experiences. Not surprisingly such an agenda meets some resistance, and this not only from those students who simply say they do not like to work socially (in groups etc.).

- The open-ended agenda of problem solving makes intellectual demands (as well as the above) which are quite different from those of AS/A level: the latter have perhaps become increasingly procedural and focussed on test items over the last two decades (during which period modelling and coursework disappeared from most of the mathematics GCE syllabus and assessment).

Nevertheless the course has shown that it is possible to deliver such a course in a first year university Physics degree course and produce a credible assessment and grading (and a statistically significant addition to measures of disposition as mentioned above).

Furthermore the best students evidently developed a modelling and problem solving competence (made most apparent in the work done by one student who is highlighted in the video on www.transmaths.org/mmps/). The two strongest students interviewed produced work that showed tenacity, and the enthusiasm and commitment to explore their own models and refinements.

Another group of students who scored well were less convincing: they said they had had the modelling cycle 'drummed into them' but their work showed some signs of being (relatively) formulaic and reproductive. Their work showed a refinement cycle (of a ball rolling round a vertical circular track) but they did not make clear WHY they had done this cycle of problem solving, except to more closely 'fit' to data collected in a practical. This is worthy and shows some insight but lacks the creativity and awareness of the best projects. One may also wonder if the approach had really become 'internally persuasive' or whether in fact they were performing as they had thought was expected.

It may be a coincidence that the outstanding students were both relatively mature: arguing that this course had offered them a truly (and unusually) challenging experience that encouraged them to work hard, perhaps even 'too hard' in narrowly strategic terms. The less mature group – though successful – seemed more concerned with 'playing the game' and a strategy of optimising grades per work-input (a model that might perhaps be measured in degree points per hour of study?) – commenting they were unsure how much they 'needed' to put in to score well enough (time being a key resource to be distributed between their various courses). Our recent research has shown how deep this kind of strategic rationale goes among todays students and academic staff.

Comments on context of the course

A number of elements of the context of the course deserve comment: the course was designed for the 'weaker' half of the cohort regarding their preparation in mechanics (i.e. those who had only done one module of mechanics at AS/A level or fewer). The logic of this is contestable: and in itself this 'setting/grouping' of students can be an important factor in shaping students' dispositions. If students need to develop MM&PS skills then it seems arguable that the entire cohort should be offered the course, i.e. some such course should be obligatory for all.

Conclusions and discussion

I conclude:

(i) The course managed to re-introduce Mathematical Modelling &Problem Solving (MM&PS) into the undergraduate student experience in an arguably 'authentic' course in mechanics in first year Physics.

(ii) The selection of students for this course may have mitigated against its communication of its aims as a modelling course.

(iii) Students dispositions (and probably their awareness) of Mathematical Modelling significantly increased between measures 'before' and 'after' the course.

(iv) The best students certainly revealed in their work a convincing grasp of modelling, the modelling cycle and model-refinement, and of the open nature of some mechanics problem solving. An example project and a discussion of how it exemplifies the intended learning is revealed in the video made for those that follow on http://www.transmaths.org/mmps/.

(v) Some other students did not benefit to this depth, and there are gradations in between 'disengaged' and a fine awareness of the full modelling cycle in the student responses.

(vi) This development work faced and will continue to face some significant problems in terms of students' dispositions and awareness; the conditions (e.g. in the School) in which the course is implemented may have a significant bearing on this. To sum up: this curriculum development will need to be embedded institutionally (e.g. in the programme aims, documents and policies) before the student cohort fully accept it.

This course stands out as being innovative and against the grain of much of the rest of the degree course (as perceived by staff and students). We know from the literature that there are plenty of examples internationally in various places at various times when MM&PS courses have been implemented and reported to be successful. The argument in favour of such courses does not rest on its popularity or acceptability by the students, but rather in the argument that 'the point' of a degree is to broaden one's understanding and competence. Even within the employability argument, Mathematical Modelling & Problem Solving are arguably more important than content–led courses. But in terms of teaching an authentic practice of Physics at first year level, and one that prepares students to become academic Physicists, modelling with mechanics should surely take the highest academic priority.

The evaluation, situated within such a belief system, nevertheless poses some challenges to those seeking to implement such practice: it is not all plain sailing and evidently will require sustained commitment over time by university academic staff and management. The appeal to colleagues might therefore best be framed as "is it worth trying to do anything less?"

References

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Lesh, R., Galbraith, P. L., Haines, C. R., & Hurford, A. (2010). Modeling students' mathematical modeling competencies. ICTMA-13. New York: Springer.

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Pampaka, M. & Williams, J. (2012). Report on the evaluation instruments piloted in the 'Mathematical modelling and problem solving' (MM&PS) HE_STEM projects. The University of Manchester (www.transmaths.org/mmps/).

Video and reports on www.transmaths.org/mmps/

5. HE STEM PROJECT SYMPOSIUM

Mathematical Modelling and Problem Solving,

(Weetwood Hall, Leeds, Monday 28th May 2012)

The aim of the symposium was "to bring together for a 1-Day meeting, all 13 project partners from 8 universities, the evaluation team from Manchester University, Michael Grove (director of the HE STEM Programme), and representatives from MEI and the Further Maths Support Programme (those concerned with Outreach), so that all could see and discuss the work in progress on the Mathematical Modelling and Problem Solving project. The programme for the symposium is given in Appendix[2]

The morning session began with the original partners presenting work done on MMPS which they had introduced this academic year into their undergraduate curriculum. The session concluded with a healthy discussion of the various projects and an examination/discussion of students work.

The afternoon session opened with all new partners outlining their plans for introducing MMPS into their undergraduate curriculum in 2012-13.

The evaluation team (Julian Williams and Maria Pampaka) described in some detail

(i) findings from evaluation studies

(ii) filming of students engaged in workshops

(iii) filmed interviews with students about the course and their project reports

Finally the original 4 partners described how their Outreach Programmes had progressed towards involving students from local schools and colleges in MMPS activities.
The meeting concluded on a high note, with encouraging closing remarks by Michael Grove and Mike Savage.

6. Outreach Day: *Mathematical Modelling and Problem*

University of Leeds, Tuesday 26th June 2012

A total of 56 sixth form students (who had just completed AS studies) visited Leeds University for a full day of "Modelling and Problem Solving in Newtonian Mechanics". The meeting, held in the School of Mechanical Engineering , was designed so as:

- To enable students to appreciate the value of studying as many modules of mechanics as possible in the sixth form so as to be well prepared for any STEM degree in Higher Education.

- To enable students to experience working together in small groups on 2 modelling projects that investigate
    i. Finding the optimum pulling angle in order to move a heavy load

    ii. Modelling the motion of Falling Bodies

These modelling projects were specifically designed by Dr Duncan Borman (Civil Engineering) and Professor M Savage (Physics and Astronomy) to

i. enable students to experience and hence understand the crucial role that setting up a model plays in converting a real problem into a mathematical problem
ii. solve the mathematical problem and get solution(s) that could be interpreted and/or validated experimentally
iii. reflect and so recognise the various stages in the modelling cycle when carrying out a modelling project

This HE STEM outreach event proved to be a highly successful first step in building an effective bridge at the transition between local schools and the University of Leeds- which will be followed up in 2013.

7. Appendices

[1] Project Partners

Original Partners (2010)	Leeds (Physics and Astronomy)	Mike Savage
	Manchester (Mathematics)	Louise Walker
	Keele (Mathematics)	David Bedford/ Martyn Parker
	UWE (Engineering)	Kevin Golden
2010/2011	Leeds (Civil Engineering)	Duncan Borman
	Leeds (Mechanical Engineering)	Mark Wilson
Recent Partners (2011)	Bradford (Mathematics)	Jon Purdy
	Leeds (Chemistry)	Annette Taylor
	Leeds (Electrical Engineering)	Bob Kelsall
	Leeds (Applied Mathematics)	Thomas Wagenknecht
	Loughborough (Mathematical Eduacation)	Paul Hernandez-Martinez
	Portsmouth (Applied Phsyics)	Chris Dewdney

[2] Symposium Programme:

Mathematical Modelling and Problem Solving

Monday 28th May

9:30 – 10:00	Arrival; tea/coffee	
10:00- 10:15	Welcome and Introduction to the Symposium	

Presentations by Initial Partners

10:10 – 11:30	UWE (Eng)	Kevin Golden
	Keele (Maths)	Martyn Parker
	Manchester (Maths)	Louise Walker
	Leeds(Civ Eng)	Duncan Borman
11:30 - 11:50	Coffee/Tea	
11:50 - 12:20	Leeds (Physics)	Mike Savage
	Bradford (Maths)	John Purdy
12:20 - 1:00	Discussion of Projects and Students Work	
1:00 - 2:00	Lunch	

Presentations by New Partners and Evaluation Team

2:00 – 3:00	Leeds (Mech Eng)	Mark Wilson
	Loughborough (Maths Ed)	Paul Hernandez-Martinez
	Portsmouth (Applied Physics)	Chris Dewdney
	Leeds (Electrical/Electronic Eng)	Robert Kelsall
	Swansea (Maths)	Alexander Potrykus
	Leeds (Chemistry)	Annette Taylor
3:00 – 3:30	Evaluation: Julian Williams; Maria Pampaka	
3:30 - 3:50	Tea/coffee	
3:50 - 4:15	Outreach/Discussion: Charlie Stripp (MEI and FMSP)	
4:15	Closing Remarks: Michael Grove; Mike Savage	