Students' mathematical identity and its relation to classroom mathematics social practice.

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The paper is based on the ESRC TLRP research project in widening-participation in HE, `keeping open the door to mathematically-demanding F&HE programmes', which aims to understand how cultures of learning and teaching can support learners in ways that help widen and extend participation in mathematically demanding courses in F&HE.

In this paper we address the following questions: how do some students talk about their mathematics social practice? And (conceptually) how can different pedagogies mediate mathematical identity?

We interviewed 50 students from 5 institutions on three occasions. On two occasions, one towards the beginning and one towards the end of the AS year, the interviews focused on their mathematics biographies and how these connect to other aspects of their imagined futures, such as going on to university. A further round of interviews was conducted in order to access students' self-identity about their mathematical practices, conducted mid-way in the academic year. In these interviews we used mathematical artefacts of relevance, e.g. pieces of work they had produced in lessons we had observed or their actual examination paper as a means to stimulate mathematical discussion.

We draw on socio-cultural theory, particularly Cultural-Historical Activity Theory, to provide a view of identity as in dialectical relation to practice (Roth & Lee; 2007; Roth et al., 2005). We analyse the interviews using expression of self-identity about their mathematics social practice as the unit of analysis. But we analyse these for their self-identity in relation to their mathematics social practice (Sfard, 2005; Holland et.al., 1998): we look for the use of cultural models (Gee, 1999; Holland & Quinn, 1987; Holland et al., op cit), and especially cultural models about mathematics (see also Nasir 2002, Nasir and Saxe, 200?), which they use as discursive tools for self-authoring during the interview event in order to detect changes in *mathematical* identity.

We provide an example of a mathematical modelling genre which can invoke a view of maths as useful and an example of a sociable/constructivist mathematics genre which can invoke a view of maths as sociable and enjoyable. We found cultural models of mathematics as 'useful' and as 'enjoyable' can provide powerful leverage in changing mathematical identities.

We found that when a cultural model about mathematics holds in *both* a student's mathematical self-identity *and* in the student's leading educational identity, e.g. in gearing towards university as a budding biologist say, that this can influence decisions to continue with mathematics. Hence, mathematics pedagogy needs to foster these 'in common' cultural models. Ryan and Williams (2007) state that '*the purpose of pedagogy thus becomes to help connect the scientific essence -, that is, the mathematics, with the everyday'*. A lemma may be, in relation to our sample, that we can concretise the reference to 'the everyday' and in doing so in effect replace it with '*practices of significance to the leading identity'*, for example, these might be with physics or chemistry classroom social practices, in the case of budding scientists *as well as* with imagined practices of the future regarding transition into work (part of being human) and hence maturity.

The Mathematics 'Problem'

Mathematics is crucial for students' access and success in a wide range of F&HE courses, especially in science, engineering and technology (Roberts, 2002). Advanced level (hereafter AS/A2) mathematics at 16-19 has been particularly pertinent historically to economic life chances (Wolf, 2002) and continues to be relevant in the 'choice making' (Raey et al. 2001) of students into and out of opportunities for numerate education in F&HE.

Recently, the Smith Report again emphasised the economic significance of mathematics and the 'Mathematics Problem' – i.e. the shortage of mathematically well- qualified students and graduates at every level (Smith, 2004). This report calls into question, not only the appeal of studying mathematics but also the kind of mathematicians we are producing: industry and science need proactive problem solvers, with good conceptual understanding of mathematics. We need to understand how mathematical thinking can become accessible to more students, especially those for whom AS/A2 mathematics is a barrier to progressing into mathematically demanding courses that confer social, cultural and economic capital. The social science literature on widening participation (e.g. Roberts et al., 2003; Mackie, 2001) suggests a positive disposition towards a field of study is often decisive in persistence with study. Fostering more positive dispositions for studying mathematics is, therefore, crucial, if we are to address the 'mathematics problem'. This is part of the solution.

Introduction

In this paper we demonstrate that students' perceptions of the classroom mathematics genre they experience can influence their decision to continue, or not to study mathematics. Here we connect with the body of research which has demonstrated that mathematics pedagogy can impact the genre of mathematics that learners experience, which has salience then when considering the kind of mathematicians being produced. For example, several authors have shown a relation between the kind of pedagogy experienced and how students think about themselves as learners of mathematics (Bartholomew 2000, Boaler 1997). For example, Boaler and Greeno (2000) found that those students who experienced an inquiry- and discussion-based pedagogy (what Morais, 2002, after Bernstein, 2000, would call a weakly-framed pedagogical discourse) identified mathematics as a field of inquiry in which they could engage actively and 'sociably' (see also Williams and Davis, 2006 and Davis and Williams, forthcoming). Significantly, and in contrast to more 'didactic' pedagogies, this encouraged certain kinds of students into further study of mathematics. For example, Mendick (2006) notes how students' understanding of themselves in relation to learning maths is connected with their decisions about studying mathematics at 'A' level.

The purpose of the paper is not then to establish that a relationship exists between the kind of mathematics experienced by learners and their view of themselves as mathematicians. This has already been done. What it does is to show how engendering a cultural model of mathematics as 'useful' to their imagined futures can impact changes towards more marginal or more participatory mathematical identities and sometimes can result in changes in decisions. We conceptualise the cultural model of mathematics 'as useful' as at the boundary between present and imagined future identities. In order to examine how such cultural models about mathematics impact students' decision making regarding their continued participation in mathematics we expand upon the idea of a leading identity (e.g. in Black et al, 2007) and argue its facility for understanding shifts towards more marginal or more participatory subjectivities (see also Davis and Farnsworth, 2007).

We show how for our sample of 50 that students taking Use of Mathematics AS course reveal a deepening understanding about the utility of mathematics compared with students taking a traditional AS course.

Definitions..something on scientific and every day..Julian, perhaps you could just write something briefly about what we mean by this please?, it will take me much longer to go through your book to find what you say.

Mathematics classroom social practice and mathematical identity.

'From a social practice perspective it is through cultural practices as people 'do life' that social identities are constructed (Nasir and Saxe, 2003). As Rogoff (1990) says 'this involves a relation between the individual and the social and cultural environments in which each is inherently involved in the others definition. None exists separately'(p.89). Understood as a recursive relationship between learner's cultural resources and classroom pedagogic culture, sociocultural theoretical perspectives enable a view of student's meaning making as embedded in their social and cultural milieu (Gregory et al., op cit., p. 86). Our conceptual framework to examine this dynamic relation between mathematics classroom social practices and mathematics learner identity is expanded upon below.

First we take a view as with (Holland, Lachicotte, Skinner and Cain, 1998) that 'identity processes are no longer seen as connecting individuals in homogenous or fixed ways; our identity work is never 'done', it is always on-going' (p.??.). Although a person's identity is not determinable, neither is the meaning-making involved in identity-work entirely free but, instead, is mediated by the discourse and practices of people's communal social activity systems (Engestrom, 1995). Situated human creativity exists not despite, but because of social structures and concrete activities (Leontev, 1978) Thus, talk about identity in social terms does not deny agency but views the very definition as something that is part of the practices of specific communities (Davis, 2007). Thus, we allow for the possibility that engagement in a different mathematics social practice can influence mathematical identity.

Second, we draw on the notion of cultural models, which Gee (1999) refers to as everyday theories or beliefs that inform action, as our tool to detect changes in identity. 'Cultural models are not static …and they are not purely mental but are distributed and embedded in socio-culturally defined groups of people and their texts and practices (Gee, ibid., p. 23). Cultural models tell us what is 'typical' or 'normal' and mediate our actions, not universally, but from the perspective of our experiences. According to Holland et al (op. cit), it is this 'stuff of existence', what is real to people and has meaning for them, which 'grant shape to the co-production of activities, discourses, performances and artefacts' (p.51). Students' identity-work can then be viewed as mediated by cultural models which they can draw on as 'self-authoring' tools as they negotiate their identities (Holland et al., ibid.; Holland and Lave, 2001). In our view, this places cultural models at the boundary between practice and identity (a dialectic) and, hence, provides an analytical tool for studying changing identities. In our view changing, breaking or replacing cultural models may afford the space for people to renegotiate their identities. As Gee notes 'cultural models …are the basis on which choices about exclusion and inclusions and assumptions about context are made (Gee, 1990, p.90, cited in Moreira, D 2007).

Thus, in our data, methodologically, we need to look at mathematics classroom social practices which offer in some way qualitatively different mathematics genres to the kind experienced previously in order to detect changes in the way students talk about themselves in relation to their social practice of mathematics. Hence, we can examine how students' cultural models about their classroom mathematics social practice change when they experience something different and how they self author themselves using these perhaps revised cultural models as discursive tools.

Thirdly, we recognise that changing identity requires that engagement in a practice (or practices) of whatever kind must at some point start to 'turn inwards'. The use of ' psychological tools ' (such as language) in practice is always double edged, what is used in social interaction comes reflexively to be used internally, on the self (Vygotsky, cited in Hernandez et al, 2007). This gives us the motive to examine students' self-identity in relation to their mathematical social practice. By self-identity we mean the reflexive 'story of the self', or 'biography' that we tell ourselves and sometimes others about our trajectory. As an analytical tool, self-identity in effect becomes the storying of the self as told within the interview event which we consider as a co-construction between interviewer and interviewee. [1]Thus, our discourse analysis with regard to examining the relation between mathematical social practice and mathematical identity is of self-identity extracts about engagement in mathematical social practice, for example extracts such as '*And you can interpret that sort of thing from the graphics. That's what I did there.', 'I'm a klutz when it comes to exams'* or '*so it[graphic calculator] is really useful, yeah. I found it…I don't know how I would have coped without it, to be honest.*'
'*where* [in maths] *you can do your own answers'* and other subjects where '*you get taught what other people have found out*.'

Lastly, Ryan and Williams (2007) articulate an important consideration when developing identity, 'A social theory of learning implicates the changing relevance of mathematics to a learner's identity as they grow up. If we accept for the moment that the leading, predominant activity of the infant is found in 'play', and of the adult is found in 'work', then schooling must provide for transitional activities and identities between those of 'play' and 'work' (p.163). Here we turn to the underpinning theory of leading activity as developed by Leont'ev (1981). According to Leont'ev, 'Some [activities] play—the main role in development and others a subsidiary one. We can say, accordingly, that each stage of psychic development is characterized by a definite relation of the child to reality that is the leading one at that stage and by a definite, leading type of activity (Leont'ev, 1981, p. 395 as cited in Griffin and Cole, 1984, p. 50).

By talking of a leading identity (see also Black et al, 2007) we stay close to the notion of leading activity, with which the subject is in dynamic relation. Particularly here we look at the concretise of the concept for education considered as transition from play to work (Ryan and Williams, op cit). Leon'tev considers play the leading activity for a child and 'work'[2] the leading activity for adults. In concretising the notion of leading activity we look to the material activities that make up the umbrella category of leading activity (an abstraction) for our sample. Main activities for our sample are their participation in their various lessons, but there are others. One of these is engagement with university activities, more specifically with transition to university activities. This may be insignificant for some because individuals may not intend to go on to university. On the other hand, others may already engage with the world of university, for example, they may have friends at university, attend open days or request university publicity brochures and or simply fantasize what it may be like to belong to a university, perhaps studying a particular subject or even being on a specified course.

If education is seen as providing the transition from play to work then transition to university is *a part of* leading activity for our sample, and thus a leading identity (defined

[1] Footnote as with ifs paper to go here
[2] Work does not necessarily imply paid work.

as leading here because of its connection with their leading activity as transition between play and work) becomes their identity regarding this transition, which is why the notion of designated (imagined future) identity (talked about in Black et al, op. cit) is important. Leading identity as we define it is related but somewhat different from the concept of 'significant' and 'designated' identity offered by Sfard (2005), and is of the 'self-identity' reflexively discursive type.

For our sample, the furthest progression they have on the path towards worker is their engagement in their transition (almost in all cases for our sample, see Hernandez et al, 2007)) to university, and so it is these imagined identities as university students and eventually as particular kinds of worker that mark a temporal boundary of their leading activity. It is this borderline of the concrete activities comprising leading activity (see also Davis and Farnsworth, op. cit) that we see influencing decision making (of the kinds described in Black et al, 2007 and Hernandez et al, 2007) that we term as leading identity (it is a more 'advanced' identity towards worker and meaningful contribution to the socio-economic structure) and is *in part* designated.

We argue that examination of this leading 'somewhat designated' identity is required to understand the *conditions* surrounding *changing identities*. Identities, as with practices, do not stand in isolation, we have many identities and engage in many and diverse practices. Crucial then is which of these are salient to the problem in hand. For this we are saying that it is the *relation* between the identity in question and leading designated identity that is of salience. Thus, in order to understand the conditions pertaining to mathematical identities undergoing change we need to look not only at what students tell us about themselves in relation to their mathematics social practice, but also at what they tell us about *mathematics* in relation to their *leading identity.* This provides for the identification of discursive boundary objects/cultural models between leading activities *and the more advance, guiding and somewhat designated identity.*

In Davis and Farnsworth (2007) we found that the utility of the identity in question (in that paper students' personal financial management identity) within the context of their leading identity (as students on their way to or having reached university) was crucial to their participation and identification with personal financial management. The concept of utility is also of relevance in our data (see later). Operationalising this in terms of cultural models about mathematics we see later in the paper that mathematics as useful for the part experienced/part imagined –part designated future identity that there is an engagement of mathematical identity with the leading identity.

Ryan and Williams (2007) state that '*the purpose of pedagogy thus becomes to help connect the scientific essence -, that is, the mathematics, with the everyday'*. A lemma may be, in relation to our sample, that we can concretise the reference to 'the everyday' and in doing so in effect replace it with '*uses of maths salient to the leading identity'*, for example, these might be practices allowing the mathematisation within contexts for biology in the case of budding biologists.

The project and methodology

This paper draws on analysis of case studies of mathematics classroom pedagogic culture. This data is part of a larger mixed methods project, 'Keeping open the door to mathematically demanding programmes in FHE.' funded by the ESRC TLRP in Widening Participation. The full project employs the case study research along with a quasiexperimental design comparing two AS programmes in mathematics, traditional AS mathematics and 'Use of Mathematics' AS.

The Use of Mathematics AS development (UoM), was introduced in curriculum 2000 and in 2005 was completed by over 1000 students, the majority of whom are in FE/6th Form Colleges, and largely in urban 6F&FE Colleges (see appendix A for data). This course was designed precisely to overcome the traditional A level barrier and to appeal to students with limited previous success at GCSE, by engaging with powerful computer technology (software that enhances mathematical power), more dialogical, formative assessment by portfolio (appealing to vocational practices), and through its emphasis on mathematical modelling (relating mathematics to its uses). Hence, this is one programme where we might look to seek alternative mathematics social practices.

However, we have always been aware that Use of mathematics and AS/A2 mathematics would not necessarily be implemented in distinctly 'didactic' versus 'inquiry-based, sociable' ways, and that classroom culture is influenced by a complex of factors, including for instance teachers' beliefs, training and dispositions. Hence, although we expected to find distinct features of pedagogic culture associated with the programmes, we also expected variation in learning contexts and recognise the need to study the full complexity of a diversity of classroom cultures.

So far we have interviewed 50 students from 5 institutions on three occasions. On two occasions, one towards the beginning and one towards end of the AS year, the interviews focused on their educational biographies, with an emphasis on mathematical biographies. By this we mean an interview about the previous and current experiences about learning maths and about school or college, which we situated socioculturally by exploring peer, family and community influences. A further round of interviews, in the middle of the academic year, was introduced in order to access better students' self-identity about their mathematical practices. These additional interviews were conducted to encourage more specific and deeper reflection on practice. In these additional interviews we used mathematical artefacts of relevance, e.g. pieces of work they had produced in lessons we had observed or their actual examination paper as a means to stimulate mathematical discussion.

For the vignettes, students from two classrooms were selected in order to offer mathematics social practices that might be expected to be different from those that most students would have experienced previously. Hence, these classrooms provide us with the potential to examine *changing* mathematics self-identities, and within these classrooms vignettes of students' mathematics self-identities and their relation with a leading identity have been selected to give examples of how the theory we have expounded upon earlier can be grounded or concretised. For the cross case comparison we draw on data from the first round of biographical interviews in order to show how the cultural model of mathematics as useful' varies between programme (e.g. UoM versus traditional AS mathematics).

These data are later used in the discussion when we address the questions:

(i) How do some students talk about their mathematics social practice?

(ii) Conceptually, how can different pedagogies mediate mathematical identity?

Two Vignettes

We present two vignettes that demonstrate how engagement in different mathematical social practices (or different genres of mathematics) sometimes reach the point where they have started to 'turn inwards' and implicate a changing mathematical identities. We also show that such turning inwards happens in different ways and this may not always be sharp or deep.

In the first vignette we consider Anthony who is a 'Use of Mathematics' student (achieving a C grade in this AS). During the course his teacher decided to introduce the class to graphic calculators. We draw on an interview in which Anthony uses his graphic calculator as he verbalises his problem solving when working through selected examinations questions from the actual papers he had sat a few weeks earlier. In addition, we draw on his two 'biographical' interviews.

In the second vignette we consider Reanne and Kirsty who are AS maths students and who are engaging in constructivist motivated mathematical social practice. Based on teachers' self-report of their 'teacher/student centredness' scale we found their teacher to be the least transmissionist oriented teacher of our larger survey sample of about 60 teachers. We draw on an interview with Reanne in which she is asked to talk through the mathematics she did during small group work in a recent lesson. In addition, we draw on biographical interviews with Reanne and Kirsty. For a more in-depth contextual description of the mathematical genre/pedagogy they belonged to see Wake et al (2007).

It should be noted that it is not our intention to compare the pedagogic practices of the two classrooms. Our purpose in the vignettes is simply to show that sometimes the mathematics social practice does go 'inwards to the self'. These self-identity expressions about mathematics practice and maths in relation to their leading identities are accompanied in the vignette within contextual oriented dialogue within the interviews which are used here in order to illuminate aspects of the mathematics discourse genre associated with each classroom.

Becoming a Mathematical Modeller.

Let's begin by looking at Anthony's mathematics practice, which we see mediated by his use of the graphic calculator. In the following extract he works with the calculator to find a visual solution to the problem which he sets up as a graph. In visualising the problem, he makes an adjustment to his conceptualisation to see the solution where by looking to see the coordinates of the graph where y=0.25. He realised that looking where y=0 would be erroneous and adjusted for this visually, much in the same way one might rework a line in an algebraic solution to an equation and instead of putting an expression=0 putting it equal to 0.25 and solving.

A: *I think I've…let's just turn it off and on again. It's died on me a bit. Change the window a bit. The window's a bit big. So if I do the minimum window at 4000 and the maximum window at 6000, let's see how that looks. There, you can see it a bit better now. So I just try and spot where it crosses the x axis and hopefully, it will give me around 5730. If I zoom in, and you can just keep zooming in on the point and it'll give you more and more accurate measurements of where the point crosses the x axis. If you zoom into a point, it'll just keep getting smaller and smaller so that you can be perfectly accurate to decimal places and stuff like that so if you're far out and you see a point then it's not very good, it's not very accurate, is it? If you zoom in, sort of thing.*

Here we note the use of the terms window and zoom, essential facilities if he is to physically find the solution using the calculator. Here the calculator is used as a machine to solve problems.

PD: *And can you get the exact point from there?*

A: *You probably could, yeah…I'm looking for 0.25 because it's saying you need to look for half the amount that it started with, the half-life of it. So I'm zooming in now. Ok.*

J: *So you're looking for the y to the 0.25, are you?*

A: *Yeah. I'm trying to…there's an arrow which is using this now. And I'm doing that now.*

J: *And it's producing the x and the y coordinates of the point?*

A: *Yeah, exactly.*

J: *So you're going to zoom…you're going to sort of run along until you get the y value to be 0.25.*

A: *That's exactly it. Yeah. Ok.*

J: *I thought you were looking for where it crossed the axis.*

A: *Yeah, that's what I was thinking.*

J: *That's not right.*

A: *No, yeah. I was thinking that. That's when it actually reaches zero. (Anthony confirms he is looking where y=0.25)*

In the extract above we see how Anthony's mathematics takes place between him and the calculator. Paper and pencil is not used and there is no recourse to algebra. He realises he needs to just his thinking and does this using the gcalculator facilities.

Suzanne on the other hand takes a different approach to the solving of the same problem.

S: *Oh, you…that one's just solving the equation. Solving 0.5 equals e to the minus blah blah blah and you have to find t by taking ln [natural logarithms] on both sides I think.*

PD: *And is that how you did it? You kind of did it using…*

S: *Like, here.*

PD: *ln, you said.*

S: *ln*

PD*: So did you do that more as a handwritten calculation?*

S: *Yeah.*

PD: *Solving the equation* [algebraic].

[… insert Suzanne's working if can get it.]

S: *This one. I think it's that.*

PD*: Right. Let's see. Then show it takes approximately 5730 years. Yeah. Is that comforting? When you find that you've got the answer?*

We see Suzanne remaining within a symbolic algebraic genre of mathematics: we include her extract simply to reinforce how Anthony is working in a different way, which we could perhaps argue to be a different mathematics genre in the same way that spread sheet maths may be regarded as of a genre (see Wake and Williams, 2006?).

Anthony on the other hand mathematises within the bounds of his graphic calculator. We can see this when he says '*Because in these questions, it's asking you to…you'd have to imagine the graphs rather than looking at an equation…you'd have to…you'd look at an equation and then liken the graph, wouldn't you? Rather than you can actually see the graph with this, you can actually physically see what you're looking at and interpret it yourself.*' In this extract we can see that for Anthony solving equations implies doing so visually through the use of graphs. Anthony reveals a graphic calculator genre of mathematics, for instance, consider the extract *And so if you played with that bit, say you did 5 plus 5 for instance, you could see how the graph changes with different values for A and B.* In this discourse 5+5 does not equal 10: the 5's refer to m and c in the expression mx+c for the equation of the straight line, which he interprets as parameters within procedures available in the calculator.

Through 'playing' with the graph he says *it's gone steeper and closer to the line so it's actually going bigger, it's going faster than before so…but if you make it lower, things are going to go…you can change it again. It's at more of an angle so you can see the B is actually changing the speed of the vehicle, I guess. Because it's changing the steepness of the graph and that's what's just changed. I changed it from, I've changed B from 5 to one and seeing the effects of that and you can tell by that what it's doing to the graph and you can interpret it by thinking about how it relates to the data that you're using on the graph. So for instance, If something's going from that to that, it's going at less speed, isn't it? It's at more of an angle so you can see B is actually changing the speed of the vehicle, I guess.*' Here Anthony mathematises as he models the problem, which we can see in the frequent references made to the speed of vehicle in relation to the graph. His reflection verifies his self- awareness that he is making an interpretation of a problem using graphics, '*And you can interpret that sort of thing from the graphics. That's what I did there.*

Given Anthony's commitment to a graphic calculator genre of mathematical modelling we can examine how this may mediate his mathematics self-identity. He goes on to say *Without the graphic calculator? In the whole exam? I don't…without a graphic calculator, I don't…I think it would be a lot harder than it was, to be honest with you. There's always something you can do with a graph or there's something you can look at and compare and anything like that with the graphs so I don't think I could do much without it, really.*

And in doing so he reveals '*I don't think I could do much without it, really*'. We see here a self-reflection about mathematical identity in practice. Anthony is someone who not only uses the graphic calculator to do mathematics, it is an essential tool. He expresses this is similar ways on several occasions during the interviews for example, '*so it[graphic* *calculator] is really useful, yeah. I found it…I don't know how I would have coped without it, to be honest.*'. However, we note Anthony's awareness that in the exam he is required to at least also use symbolic algebraic maths*, 'actually, if it said to calculate something, I'd calculate it and check it with the graph, not actually use the graph to tell me what the answer is'.*

It should also be noted that there has been a shift in Anthony's mathematical identity in relation to the graphic calculator. In his first interview he said '*I think at the start the graphic calculators, I had never heard of graphic calculators before. It was just drawing graphs but here at first I was like "oh god it's a big calculator really" I don't know what to press! I didn't know how to use it, but now it's straight forward once you get used to it.*'

Anthony articulates that he sees Use of Mathematics as being different to '*normal*' mathematic.

A: *Well, maths as I see **normal**, is you learn something and you just do it, you know. You just learn it and you apply it to a question. Whereas in use of maths you use it and you apply it to a certain situation rather than just a question, if you know what I mean.*

MP: *Yeah.*

A**:** *So it was quite a different way of learning and quite a different way of teaching, really, but once you got used to it it's quite…I think it helps. …Well, I guess it's just the way it's done. ….because when you're taught it you're not just learning it, are you? You're learning about how it's applied to a certain situation and not just how to do it because I think, like I say, normal maths is just learning how to do it and how to do this. Like, for instance, multiplication. You learn how to do it and you apply it to a simple question. You don't look deeper into it and look at how you could use it in real life which is what use of maths is all about, really, isn't it?*

Anthony picks up on mathematical modelling when saying '*how it's applied to a certain situation and not just how to do it',* although here he does not here use that term. For Anthony mathematical modelling provides him with a mathematics for 'everyday' *use*. With reference to mathematics classes as distinct from UoM classes, Anthony says *Because when you just learn it, you're thinking, when am I ever going to use this? When am I ever going to need this? This is boring, this isn't very good. But when you apply it to a situation you think more about it and you can understand it more because you know what you're doing with that and what you're doing to it and what you're doing with the situation and how to use it and things, which is better, worth learning I think. I think I'd prefer that, better than further maths anyway.* This suggests a (perhaps at one time) mathematical identity that considered mathematics as not of use and this lack of use (utility) he connects with a view of mathematics as 'boring'. On the other hand, Anthony connects application with understanding and relevance, '*worth learning'*.

When asked why he had chosen to do mathematics he said '*I thought it was going to help me with my physics*.' In spite of his new understanding about mathematics as for the every day, he is less sure about its use in his biology degree. He says *I want to get a part time job so it could help with that…. Like say, if I got a part time job in a shop or something like that, it'd help a little bit and things like that but I don't think I'd use it as much as like I say, some other people might use it. Some of the people that really wanted to use and really need it, to carry on.* We see here that Anthony does not consider his mathematics essential to his own leading identity as a budding biologist**.** However, despite Anthony's changing and positive mathematical identity he has now dropped mathematics, after achieving a C grade in Use of Maths. Anthony took Use of Maths by choice as a support for his main subjects, especially for physics, and within that view Use of Maths has done its job for him. We see his view about maths as non-essential to his trajectory when he reflects back on his mathematics course: *Not doing it. Obviously I'll miss a lot of the people that were in here and stuff like that but I think it takes a weight off…personally, it would take a weight off all my other subjects and it…obviously it was good to do it for a year but then next year it'll be like, really concentrated on my main subjects, the ones that I really need to pass and things like that so I think it's better for me to drop it because of those other subjects that I really want to concentrate on.*

Our **analysis** of Anthony's self-identity about his mathematics social practice and its relation with his leading educational identity to become a biologist we note that he demonstrates a shift in mathematical identity so to understand mathematics as for the everyday. This we put down to his engagement with the graphic calculator, which became an essential tool for mathematical modelling (a new practice for him). Engaging in a social practice of mathematics of the everyday (i.e. of 'realistic' but perceived to be purposeful problems), or in other words in *mathematising* (ref) has fuelled Anthony to replace his former cultural model of mathematics as *useless* with one of *useful* and of being *boring* with being *worth learning*.

However, to understand Anthony's decision to drop mathematics (and hence end his participation in formal mathematics classroom practice) we have to look at the relation between his cultural models about mathematics and his leading identity. He perceives mathematics as useful, but not especially so in biology. Mathematics is not a subject that he *really needs*. His cultural model of 'mathematics as useful' is not aligned specifically with biology, it is not a cultural model of mathematics is useful for biology, though he holds to its usefulness in relation to physics. Indeed, his initial decision to take Use of Maths evoked his model of mathematics as useful for physics. *'I thought it was going to help me with my physics'*.and in turn physics is an essential, to be continued in A2 in order to fulfil his goal to read biology at University.

We also note that although Suzanne and Anthony were in the same class and so are party to a common pedagogy they do not both develop the same changes in identity. We see that Suzanne stays with the symbolic algebraic genre with the graphic facility on the calculator as a check and though she enjoys to learn about different applications of maths, she could not be said to exhibit a graphic calculator genre of mathematics in the way we might argue (more forcibly if space allowed) in the way of Anthony.

We should also note that Anthony did not produce many self identity statements of the type 'I am'. His only one was 'I am a klutz at exams'. However, we see in the kind of data produced his self-identity coming through implicitly in his talk. This appears to be more typical when students talk about themselves in relation to curriculum subjects, and we note a difference in this respect when students talk about themselves in relation to their futures (see Davis and Farnsworth, 2007 also). As was typical of much data of this kind self-identity expressions such as 'I am' were low frequency, though significant when they occur.

From me to we work together – a sociable and invented mathematics

Reanne had been working in a small group and solving the equations as part of a group closed-type of investigation activity (ref). The investigation was closed in as far as particular solutions were expected and the activity was designed to guide the students towards the desired 'correct' solutions'. Students had been given a number of equations which had been worked to their solutions in symbolic algebraic format in a step by step way. All the equations and their equivalence had been mixed up and cut out so they could be easily manipulated. The job of the group was to sort out the equations and their solutions. The groups had been given graphs of the three main trigonometrical functions sinx, cosx and tanx between -360 and +360 degrees as an aide. Reanne solves trigonometric equations of type reducible to quadratics without the use of a graphic calculator that Anthony had come to view as essential.

R: ***We*** *started off where we had to like, work out the first equation and you could factorize it or it was like the quadratic formula which was what we did.*

PD: *Why did you do that?*

R: *I found it easier personally. Some people like to factorize but…and then once you worked it out you …. then work out the degrees, what degrees and if they have a solution or not….* ***And then we did it*** *for cos and tan as well, worked out the degrees and things*,

PD: *So why did you do it for cos and tan as well?*

R: *Just to see if there's any like, difference in what it made. Really.*

P: *Yeah. I thought this was an interesting one. What happened when you got that answer there? Sin x equals minus 0.3333. do you remember*?

R: ***I don't know, Robert did that one. But it came out as that.*** When you type it into the calculator it comes out as 199 degrees.

We also witness frequent use of the term we, which we take as indicative of her affinity with her 'small' group. We see the sociality of the mathematical social practice see experiences (see Wake et al for more on this), which she reveals when saying ' *I don't know, Robert did that one.*'

Reanne talks about the pedagogy:

PD: *Then tell me why do you think you like using this sugar paper approach? What do you think it is?*

> R: *'it's just a bit more fun really, rather than writing out questions and all that kind of stuff. It's like,* ***all the writing in big pens and sticking it down*** *and* ***working together*** *and that.* ***I like doing stuff like that****.'*

PD: *So which part of maths, or parts of the lesson, do you like the most?*

R: *Well, I don't know. I guess learning it really, or trying to learn it. Like, when Susan's talking and things and trying to understand it. What I don't like is when we're doing it all together, like Sarah's* [teacher] *talking to the whole class, and then she'll like, point people out to like, answer a question and I don't like that because I'm frightened that she'll ask me and I won't know what it is.*

PD: *Do you prefer it, does she sometimes use the whiteboards? So that you show your answers and then you don't have to be the person speaking?*

R: *Yeah, like sometimes we work in groups of four and she'll say like, we all have to discuss it and then somebody'll go up and move, like match something up and then somebody else has to explain it. So if I don't know how to explain it, I'll go up and match it.*

PD: *Right.*

R: *And the other person will explain it*

PD: *So how does that work? Can you always, can you more or less guarantee that you can manage to do that, match so that you don't have to…*

R: *Yeah, unless like, Sarah picks you. Someone to match and someone to explain. But we all work together really so we can like, if we work together then we mainly can all explain it.*

Reanne was not alone in feeling comfortable using a 'sugar paper'/small group work approach and being aware of the cover it provided her from the whole class approach which she contrasts as personally threatening in some way. Though having observed these lessons we know that white boards and other approaches to minimise this perceived exposure are a routine part of the teacher's inclusive pedagogy. For example, in an interview with her teacher : put extract here if time.

Reanne articulates a view of a sociable mathematics, and for her this gives 'a way in' as this makes 'maths fun' and also offers some protection from exposure of her misconceptions in class and hence she associates a sense of safety about their group activity. She does not go as far as to articulate a constructivist oriented mathematical identity, despite experiencing a constructivist oriented pedagogy, although she does show awareness that her mathematics lessons are sociable, from example through her use of 'we'.

When Reanne tells us about her plans she reveals that she considers maths to be needed to achieve her future dream: *I would like to move to California when I am older.*
I: California? Like in States?
R: Yes.
I: Where does this come from?
R: Just a dream I have for a long time.
…
R: I need maths and the other subjects I am doing but I think when I go to university to be an architect it will all be in one.

Reanne see mathematics as a subject that will help secure a place on an architecture degree course. The use of mathematics is in its future exchange value. We note how this differs from Anthony's perspective about mathematics as connecting with the everyday.

On the other hand, Kirsty looking back on the same approach with the same teacher a year earlier reveals more than a view of maths lessons as sociable and articulates a sociable constructivist genre, which she does not hold about her other subjects such as chemistry or biology.

I think it's ***coz maths can be taught in loads of different ways*** *whereas chemistry and biology and things like that its text books isn't it its like fact and things and you don't just give anything by yourselves you get taught what other people have found out and it's the* *same with business studies as well, you talk about theories and stuff like that and the only real projects we have done was when they found like a product and stuff and then said to write out some things about it and stuff like that like strengths and weakness* ***whereas like maths you can do your own answers can't you*** *so I think it is different but they should learn from it.*

In stating that *maths can be taught in loads of different ways,* Kirsty is referring to there being different pathways to a solution, We can see her constructivist view of mathematics mirrored quite clearly in this extract when she makes a contrast between mathematics '*where you can do your own answers'* and other subjects where '*you get taught what other people have found out*.' She reiterates this again '*And like not only you think for yourself but like we can ask other people why they got that and it's not just black and white, like you get to a different way to work it out*.' To give another example, '*yeah that's what I like as well, you can do it one way somebody else can do it a different way but you can still be both right and that's what's good, you find your own way. It is like a bit of independence as well I think in maths whereas other subjects you are taught what it is and everything whereas maths you will go off and find what you want, it's good'*. And alternatively, '*maths it's like really good, I didn't like I would enjoy it at all, I thought it would be boring, like at school it was pretty boring sometimes but, it's really good*.'

Katie also reveals she considers maths lessons to be sociable, and notes this as different when compared with other subjects: *I really made good friends with Jay and my other friend and then there's other people that you just see in college and you talk to them about the lesson or stuff like that and cos we move around we're always talking to different people, so it's quite good, you always know the faces and stuff like that, were in other lessons you don't even know them you don't even know they're in your lesson, so it's really good.'*

Kirsty considers maths to be helpful in general. *It'll help me… just help you keep like quick with everything, cos you gotta think, you don't have to rush it and stuff, you gotta keep your mind on the ball and it helps with everything that you're doing, even like quizzes on TV and stuff cos you mind is used to be quick, and then it with that I think, it keeps you fresh, I think when you are on like the six weeks holidays and you brain goes to much you're just sat there and I get really frustrated and bored.* Her intention is to go to university to read Biology for Business and she is not expecting this to be heavily mathematically demanding.

Our **analysis** of Reanne's self-identity about her mathematics social practice and its relation with her leading educational identity to become an architect we note that she demonstrates a shift in mathematical identity so to understand mathematics sociable and fun, and as a subject needed to gain access to her desired university course. We note that Kirsty revealed that 'maths as boring' had been replaced with 'maths as really good' and as 'useful for the everyday'. Maths as 'black and white' has been replaced with 'in different ways'. Reanne articulates a view of her maths lessons as sociable, whereas Kirsty articulates a view of maths that in broad terms could be described as constructivist – oriented.

It should be noted that both these students had low to medium intermediate GCSE mathematics grades. Kirsty had resat maths AS and gone from a U on her first attempt to a B second time around. She was interviewed both before and after achieving the B grade and had been positive about the mathematics practice when hoping for a D or an E grade as well as after her more recent success. Kirsty told us that did she not feel ready to go to on to university having already spent three years at the college that she would want to continue to A2 maths. She said she would like to do A2 for herself in the future.

We note here how for students, especially those who are less confident about themselves in class, that group work can facilitate sociality in lessons, and that this can be a key prerequisite to a view of the lessons as enjoyable which can impact on more positive mathematical identities (see Davis and Williams, forth coming, but also note the work of the many who have been working on the development of more inclusive pedagogies within the field of education more generally who are too many to mention).

Changing cultural models about mathematics

We see in the vignettes examples of where students' mathematical identity has changed through their engagement in a mathematics social practice that, in some significant way or another, is different to that which they had experienced before. For Anthony we see 'useless maths' being replaced by 'useful maths' and 'boring maths with 'worth learning'. For Rebecca we see a strong sense of maths lessons becoming fun and sociable and for Kirsty, in addition, maths is no longer 'black and white' but 'negotiable' which affords her an agentive position. For Kirsty maths 'as boring' had been replaced by maths 'as good'.

All three students intend to go on to university to study biology, biology & business and architecture. Only for architecture is mathematics perceived as a necessity and this gives Reanne the impetus to continue into A2 mathematics. We see then that despite experiencing alternative mathematics social practices and altered (more positive) mathematical identities, that, for these three students, the decision to continue with mathematics was based on its perceived necessity in relation to achieving their educational goals. 'Necessity' was either decided by university entrance requirements or by the perceived utility of mathematics in relation to their future educational focus. A cultural model of mathematics as useful then seems implicated in their decisions about continuing to study mathematics, and we can see this when it occurs in both mathematical identity **and** in leading (part designated) identity in relation to transition to university.

From useless to useful maths a cross case comparison

If we now look across the project case studies to the 50 students we see '*maths as useful'* as a significant cultural model for a genre of mathematical modelling, which depending on its relevance with students' leading identity, can support a decisions to continue with maths, or can justify decisions to drop it.

If we turn to engineers taking the 'Use of Mathematic' we see they already hold cultural models about the utility of mathematics for engineering. For example, Joseph told us ***'I'm not brilliant, I am just good****. Well hopefully, as Maths is quite important I will hopefully get better by the end of the year and understand more,* ***it is going to be a big part*** [of his future studies] *because it is almost in everything, electronics as well.'* Similarly Paddy says *'I think maths relates quite a lot to mechanical principles cause I did a lot of equations there and you have to know how to do equations before… evolving to mechanical principles'*. Mohammed puts forward a similar view point which we found dominant for this group of students: *'Maths is the main part of electronics. Because to work out what is wrong, you have to work out the formula.* ***You need Maths to calculate stuff****, to calculate voltage to current. You need lots of Maths to do that as well. So my Maths course right now is helping me to do my electronics course as well.* ***That's why if I don't do Maths I can't do electronics****.'* For these students, typically they have a positive image of their capabilities. It was not uncommon to find students talking about being good at mathematics and they tend to begin the course with a cultural model about mathematics as useful (see Hernandez et al, 2007) and Use of mathematics reinforces this for them.

On the other hand, students such as Adam taking various A levels tend to begin with a view of mathematics as not particularly useful, but that typically this changes this cultural model is replaced with a deeper appreciation of various applications of mathematics, and that for some, like Adam this runs deeper and he is able to do as Ryan and Williams (op cit) argue is necessary and connect the scientific essence -, that is, the mathematics, with the everyday'. Alternatively, Anupreet also makes this connection '*It gives you a better chance of seeing **real maths** and how it works with graphs, gradients and tangents. You can see how it **is really**. It helps you to understand it better*.'

Although some students did not articulate this as clearly as Adam or Anupreet, Use of Maths students repeatedly talked about having an appreciation about applications of mathematics in positive ways and as something that makes mathematics enjoyable. For example, Suzanne explains that '*with pure maths, you don't like really learn the logic behind it. You just have loads of formulas and you don't like.. learn which scenarios to use it* [it can be used in]. *I like the idea of graphs and sorts of things and you learn about it more and I don't know really. It's different to maths.* Hence, we suggest that mathematical modelling can influence students appreciation of the relevance of mathematics.

We also note that for students on the AS mathematics course their continuation to A" is because of its perceived need. We note that typically these students do not hold an appreciation about mathematics as useful, apart from it being a means to an end. For example, like Reanne, Darren mathematics as useful does not emerge in the interviews. He states that he needs mathematics for university: '*Yeah. I probably won't study maths at university, something to do with sport… I'll obviously need my maths A level..'.* An absence of an appreciation of applications of mathematics was typical for this group of students.

However, it was amongst the AS level mathematics students we found some wanting to continue with mathematics. We note based on the wider survey data that of the 1700 or so taking either mathematics and Use of Mathematics that around 80 reported an intention to pursue mathematics at university and that these tended to be those students with higher GCSE grades, *and so* (given the way that Use of mathematics is positioned against traditional AS mathematics curricular) were almost all taking AS maths. Daniel for example thinks he will do engineering but he would also consider doing mathematics. In relation to use of mathematics he shows he sees mathematics as not a practical subject when compared with science. He says '*there's not really any kind of practicals* [experiments] *you can do in maths.*' He says '*Ah. I like both'* and would consider doing mathematics (but would also considering taking engineering) because he likes doing both the practical and the non-practical (ie. mathematics). We found little to suggest that Daniel has connect the scientific essence -, that is, the mathematics, with the everyday.

We provide a table of showing the proportion of students who identify mathematics as useful, by programme. We should note that this uses our currently best available data using the first round of biographical interviews which have been coded in ATLAS for the wider interview sample of 50. (See Hernandez et al for more information about how the data was coded).

	Maths as useful, total number in sample, %
Use of Maths, BTEC engineering students	count, count, %

Use of Mathematics, general education

AS mathematics

However, it is beyond the scope of this one paper to ground these claims here qualitatively, and all we can do is to simply point to data of the few featured students in this paper and to illustrative cross case comparisons.

Conclusion

In returning to the research questions we see that students talk in various ways about the classroom mathematics social practice they experience. We found cultural models of mathematics as 'useful' and as 'enjoyable' can provide powerful leverage in changing mathematical identities.

We found that when a cultural model about mathematics holds in *both* a student's mathematical self-identity *and* in the student's leading educational identity, e.g. in gearing towards university that this can influence decisions to continue with mathematics, this for example being evident for our engineering students. We note that this holds for Kirsty, whose decision to continue with A2 was deferred because of going to university (which she did not consider needed maths). For Katie mathematics was a personal interest, in some ways more of a hobby. We also note that a view of mathematics as useful/as part of their leading identity does not imply a continuation with mathematics as a formal subject e.g. Adam, whereas a view of mathematics as a necessity to take a particular university course can give the impetus to continue.

We note that that the decision to continue with mathematics is a complex one, it is not simply about enjoyment of a subject, it is also about its perceived transition towards work, for our sample the transition to university. We note how Hernandez et al, 2007 demonstrates that decisions to take mathematically demanding subjects at university are socioculturally influenced, as demonstrated by the identification of Discourses (in Gee's sense). However, we also note the potential power of the cultural model mathematics as useful in encouraging continued participation in mathematics. This will become the focus of future work as the project progresses. It is our intention to develop this work for a paper that will be part of a symposium at AERA2008 and in subsequent journal articles.

We also found that a curriculum such as Use of Maths with gives emphasis to mathematical modelling influences views about mathematics as useful. Indeed we note that it will be important in future analysis to unpack the cultural model 'mathematics as useful' as a means to capture students understanding of mathematics as for the everyday and how this is mediated by pedagogy.

Ryan and Williams (2007) state that '*the purpose of pedagogy thus becomes to help connect the scientific essence -, that is, the mathematics, with the everyday'*. A lemma may be, in relation to our sample, that we can concretise the reference to 'the everyday' and in doing so in effect replace it with '*practices of significance to the leading identity'*, for example, these might be with physics or chemistry classroom social practices, in the case of budding scientists *as well as* with imagined practices of the future regarding transition into work (part of being human) and hence maturity.

We conclude that to encourage continued participation in mathematics curriculum design and pedagogy needs to encourage the establishment of cultural models about mathematics that are also significant in students leading identity. We expect this to have implications for policy and practice.

References