Participating differently in mathematics: the value of mathematics, learner approach and Programme context

Davis P., Pampaka, M., Williams, J.S., Wake, G., Nicholson, S., Hutcheson, G., Hernandez-Martinez, P., Black, L. and Kleanthous, I. (2008c).
Manchester University.

TLRP-ESRC '*Opening doors to mathematically-demanding programmes in further and higher education'*, working paper.

Abstract

A cluster analysis of students discourses by codes applied to their interview responses leads to (i) categories of 'maths values' (exchange-orientated; use-orientated, and 'mixed' orientation'), (ii) categories of 'learning approach' ('surface' and ' not surface'), and hence to hypotheses about the relations between these two constructs. We find that there is a weak relation between 'surface' and 'exchange', and 'mixed' and 'other', but that 'use-oriented' and 'not surface' are related. However, these relationships are complicated when Programme (BTEC vocational-'AS uses of maths' and 'AS maths) are taken into account. Finally, we interpret these findings as patterns in discourse in their pedagogic and curriculum contexts. We suggest that when students are in dialogue with a 'uses of mathematics' Programme/ course narrative a space is created that some students may take up to become a different kind of mathematics learner – a mathematical modeller.

Introduction

This paper considers the influence of curriculum/pedagogies on students' engagement in mathematics learning, which we take as going beyond the doing of maths to include its place in their figured worlds (Holland et al, 1998) of aspirations, decisions about university and future careers. Its focus is as much methodological as substantive, for analysing the influence of pedagogy for a data set of 48 students who were each interviewed up to four times using broadly speaking narrative interview approaches, demands cross case comparison of some kind if all the data is to be considered in some way. Yet to model narrative style text is unusual, because the data is messy and there was no simple formula available to follow. The model we constructed used cluster analysis and was the result of a process of making numerous judgments and methodological decisions.

The paper begins by giving some explanation about the motive for the analysis. We show illustrations of how students sometimes value maths for the delayed gratification they believe it will afford them. The process of preparing the data for cluster analysis is then presented in a methodology section. The paper then moves on to present the results of two cluster analyses. The first is a construct of the relation between learning maths and aspects of students' educational lives and we term this construct "exchange-use". It captures how students use maths in contexts outside the mathematics classroom and their expectations for using mathematics in the future, including whether its value lies in its potential future exchange, e.g. as a credential that may help them get ahead because they believe it will look good on their c.v. (see Williams, 2008). The second constructs students' approaches to learning mathematics, and is concerned with the depth of their learning/participation in maths (Marton and Booth, 1997).

These models were each formed with the aid of Two Step Cluster analysis, which identified groups of similar cases within a dataset, i.e. groups of students with similar patterns of codes in their interviews, which represent similar views with regard to their identification and participation with mathematics. Establishing a sound basis for this categorisation as grounded in the dataset is important because this shows that there are distinct ways in which our interview sample spoke about the value of mathematics and their approach to learning. We then use these categories in order to explore how ways of identifying with mathematics (cluster membership) varied with students' programme: AS mathematics, AS "Use of Mathematics" and BTEC engineering & AS "Use of Mathematics". We then assess this approach in terms of its capacity to detect the influence of curriculum and pedagogy on students' categories.

In doing this, we draw on TLRP-ESRC project, '*Opening doors to mathematically-demanding programmes in further and higher education'*, (www.lta.education.ac.uk/TLRP.html). This project explored 6fFE students' dispositions for further study in HE, particularly in mathematically demanding subjects. As part of this project, serial interviews with 40+ students were held that sought to explore (i) the students' trajectories of their learner identity (including aspirations and choice-making, and the value of mathematics to them etc), and (ii) their experience of mathematics learning in practice, and, hence, the influence of curriculum/pedagogies on subjectivity and identity. The students included a high proportion of those from low participation neighbourhoods, first generation to HE, and minority ethnic heritages. They were all taking a level 3 course in mathematics as part of their programme. We address the questions, *in what ways do students group into different kinds of mathematics learners?* And, *how are these ways of identifying/participating in learning mathematics mediated by curriculum and pedagogy?*

Values and Learning Approaches

Readings of the interviews revealed students identifying with mathematics in different kinds of ways. This is illustrated here with regard to the value of mathematics in students' lives and their approach to learning maths.

Value of mathematics in students' lives

The following extracts show students drawing upon the exchange-value of mathematics.

"I just chose Maths because I thought it would look good on my CV and I found it easy in Year 11 and I thought why not. That's why I chose it." (Louise AS Mathematics, General Ed Student)

PHM: *Are you expecting there to be much Maths in your degree then?*
J - *No, not much. I doubt there will be much at all. As I said I think it will show that I've got good range of abilities.*
….. but I guess I used to like feeling, this sounds a bit rude, but feeling like I was ahead of people, it made me feel clever and stuff. I never really found Maths fun. I just always knew that it was good to learn and good to know about. (Joseph, AS Mathematics, General Ed student)

PHM: *If I say to you maths is hard, would you agree or disagree?*
M: *I would say maths is hard to get hold of but it's worth the struggle, but it will get you into high paid, high ranking jobs …* (Martin, AS Mathematics, General Ed Student).

On the other hand, Vladimir's talks in a different kind of way and emphasises the use-value of mathematics

MP: *How do you find Maths to be related to your degree? To the electronics?*
V: *It is very helpful, because the electronics is Maths. If you know Maths it is much easier to learn about electronics, how circuits work and make your own circuits. Usually when we present some circuits in the electronics room and you usually get sine wave and using Maths you can calculate the voltages, sine maths equations and that is really helpful.*
MP: *You like this connection.*
E: *Yes.*
MP: *Do you like what you do here is related to the course?*
E: *Maths is essential to the choice.* (Vladimir, BTEC Engineering)

Approach to Learning Mathematics/Participation in Mathematics

The following extracts illustrate some differences in emphasis between students' talk about their approach to doing maths. For example Martin emphases a procedural approach to learning and he explicitly denies conceptual understanding (of 'why').

PHM*: Ok. So this thing of going increasing and maintaining yourself within the region, does it have an explanation in terms of the problem itself? The chairs and the maximising the…*
M: *I'm not entirely sure why we used it. I can't quite remember. It's just a way "my teacher" used to try and find it. Can't remember why because I mean, somehow x is equal to y so it's just a basic line that he used.* (Martin, AS Maths).

Joseph and Louise, respectively, talk about logarithms as being disconnected from life and about procedural rule following (in this case, having exchange value, in 'passing tests'):

LB: *You don't have to read it out. Just try and explain why you do it, what the purpose is. If you know.*
J: The purpose of the logarithms is so that I can pass maths.
LB: So that you can pass maths.
J: Yeah.
LB: That's a good answer.
J: I'm not quite sure how it'll be used but as I say, the…I think they did it yesterday, the last lesson as well and I wasn't there for that so. (Joseph, AS Maths).

I: *So what did you have to do in this one then?*
L: *It's basically what we were doing before where you're given shortcuts like, the logarithm and we know from our rules that if it's together, if it's timesed, then it can mean the logarithm of each individual one added together* (*Louise, AS Mathematics).*

As Martin & Booth (1997) suggest, procedural rule-following is often associated with short-tern test passing in what they call a “surface approach”, to learning. Thus we hypothesise that patterns of surface learning might be associated with students’ ‘exchange value’ orientations. However, these students’ talk appears to contrast with that of Mahmood’s who explains how he uses logarithms to solve a problem. In the extract following, Mahmood may still be following procedures but he is practising using maths for a purpose, and is able to articulate why he is using logarithms. He thus demonstrates a connection between a ‘conceptual’ approach and ‘using’ mathematics:

Mahmood BTEC Engineering: *I had to linearise it so I could find the model or like the model or like the formula for Autograph. To do that, I had to linearise it and then find the gradient of the linear function. Then put that in the model.*
PHM**:** *How did you linearise it?*
M: *I have to write another table for…and then plot that against T and then just do a graph.*
PHM: *So these are the ones? These ones are these ones? Is it? And then how you pass from this one to this one?*
M: *I have to take another table, the length of I and then of this for each one.*
PHM: *So you apply the logarithm.*
M: *Yeah. And then once on the Y axis, I put is the log of I. On the other side I have to times it.*
PHM: *And then you graphed these ones here?*
M: *Yeah. In a straight line.*
PHM: *Obviously you get a straight line.*
M: *Yeah.*
PHM: *And how do you find the formula?*
M: From that I find the gradient.

Other students also tended to talk about using mathematics as a useful tool. For example Alan, BTEC engineering:

MP*: How do you find this connection between maths and…*
A: *Like, what I’m doing, going to be studying is telecommunications and the communication part of electronics and really communication is signals and my first assignment for this was electronic signals so that connected really well actually.*

Or, alternatively, Craig, General Ed, AS “Use of Maths” showed how he can interpret a graph as being meaningful in a given a realistic context:

JSW: *Go on then. Just give us a brief description of what it says then.*
Craig: *Ok. Well, that shows me that this substance decays a lot faster than carbon 14. It’s not…is it an asymptote? I’m not sure. Find out. Yeah, you can check if it’s an asymptote with this function which I don’t think it is. It could be. But yeah, it decays a lot faster than carbon 14 and that’s pretty much all you could say about it, really. There’s not much more you could say about it, really.*

However in the case of Jean, a General Education, AS Use of Mathematics student who will be classified in the “Surface Approach Cluster”, we can see that she recognises that maths has real world application, but her talk in this extract indicates a contradiction because she also claims that maths is not related to real life:

MP: *...the explanation is not like that?*
J: *No because you have to like talk about everything you do... I do that and then you talk about why you did it and relate it to the real world or the situation you are given, which I can’t do coz I’m used to maths being like equations and me having just have to work out the equations instead of just like relating it to real life....*
MP: *so you separate maths from every thing else?*
J: *yeah, that’s what I think of maths of like....... it’s like a science. Well... it’s not like science, but you just do it to get the right answer you don’t do it to relate to anything that you do in your own live.*

We noticed that for some students, maths is talked about as disconnected from situations, as in Joseph’s response “ *the purpose of logarithms is to pass exams*”, when he seems to draws on a discourse of exchange, possibly as a way to deflect a question he couldn’t answer when asked about the reason for solving a particular maths problem. On the other hand, other students showed awareness of mathematics as having use in understanding situations or problems. These different ways of talking about maths motivated the analysis presented in this paper because we wanted to know if there were distinct ways students engage with maths, and if the distribution of such ways varied with programme.

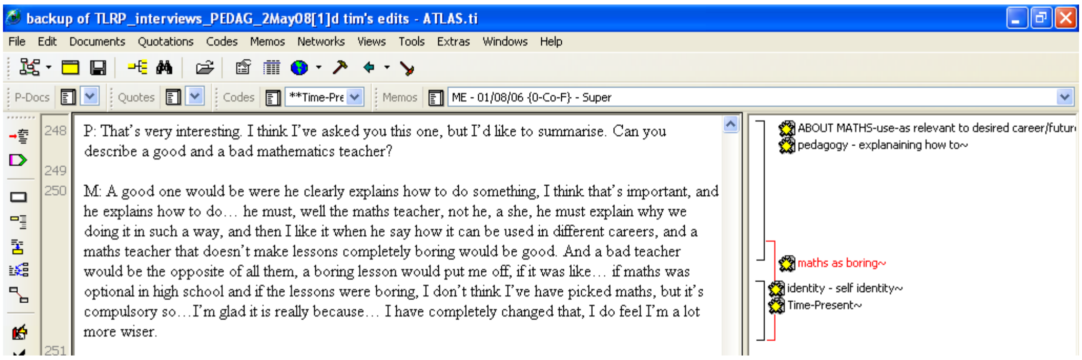
Methodology

The focus on the influence of curriculum/pedagogies on the quality of students’ engagement implies making comparison of some kind, either between students and/or between pedagogies. Cross case analysis implies comparison between cases on a number of aspects that scope the domain of interest (in our research, engagement in mathematics). Decisions about which comparators to include were taken as part of an inductive process involving playing with the data in order to find good discrimination between cases (students). This is essentially craftsmanship, which brings a sense of the artisan at work, sculpturing the data to reveal its internal structure, and the resulting clusters are the result of such constructions. An early task was to gain an overview of the distribution of students/interviews that had been assigned the codes and to examine associations between these. For example, codes such as “maths as fun” or “maths as not fun”, whereby a code of “maths as fun” implies the student told us that they experienced maths to be fun, and a code of “maths as not fun” implies they told us that they did not experience maths to be fun, enabled an examination of the proportions of students reporting particular points of view, along these very simple lines. This yielded that students who talked about learning in small groups were able to articulate that they experienced their maths lessons as a sociable experience ($p<0.05$). For ease, this analysis was conducted in SPSS after transfer of the data from ATLAS.ti. We were not unduly surprised, given we are dealing with complex subjectivities, that dissecting experiences of learning mathematics in this way produced limited results, apart from to ascertain that by and large there were few obvious patterns in the data.

However, this was an early stage in becoming familiar with the data, and these explorations of *content* were important in providing heuristics to guide the process of building the constructs.

Themes to do with the value of maths in imagined careers, in other lessons, in the everyday, as a credential that can be exchanged for something else, e.g. a better job as something worth doing even if it is hard or not interesting, emerged in interviews across the data set and at all data points, where relevant quotations had been coded in ATLAS.ti. In addition, almost all interviews at data point two (mid AS year) included text on learner approaches to mathematics, so enabling cross-case analysis of emergent themes to do with learner approach, e.g. memorisation, procedural and conceptual understanding, for this data point.

A thematic coding framework had been developed accordingly; with open "emergent" codes such as 'coursework', 'technology', 'memorising', 'uses', 'learner-approach', 'pedagogy', 'aspirations', and 'maths-values'. Although this initial framework provided for the identification of emergent themes in the data, it did not have the capacity to establish whether there were clusters of cases with distinct ways of engaging with learning mathematics (different kinds of mathematics learners),. See Figure 1:



Later this system was developed axially with a binary level of measurement for the systematic comparison of students in a number of aspects, so to be able to construct a framework suitable for identifying categories of students. Such analysis can be approached in a number of ways, and usually people choose to use spreadsheets or other databases as tools for this task. The advantage this analysis offered was that if we could ascertain that distinctive ways of identifying and participating with learning mathematics existed within our dataset, then we would have a grounded categorisation that could provide a basis with which to consider pedagogic influence on learner engagement. Our approach to the examination of the influence of curriculum/pedagogy on engagement in maths was, then, to identify groups of students telling us similar stories about their relationship with mathematics and then to consider how membership varied with programme: AS mathematics, AS Use of Mathematics and BTEC Engineering with AS Use of Maths.

We decided to try cluster analysis, given its purpose is to identify similar groups of cases within a dataset, based on case responses to a set of variables. We note that some packages such an Nvivo include a clustering/cluster analysis facility, but we had chosen the ATLAS.ti software for this purpose and so a laborious process of transfer into SPSS was involved for the set of comparator codes. This is worth pointing out because usually methodologists describe ATLAS.ti as not suitable for this purpose, but we found transfer to be possible.

Preparation for analysis involved developing the coding system so that each potential cross case comparison code had a NOT equivalent, for example "maths is useful in everyday life", and "maths is not useful in everyday life", or "maths is easy", "maths is not easy", maths is challenging", maths is not challenging", so we could distinguish how the student told us that they experienced maths. Thus, a students' identity/participation profile could be represented as a series of indicators, a string of 1s and 0s. This distinction was employed because it provided for the simplest quantification of the data; at the same time both the new codes had meaning qualitatively within their new grouping.

Given the low frequency of many of the codes some were combined. For instance, a new code "Hard" was made by merging sub-codes of "maths as difficult", "maths as hard", "maths as tough" etc, and "NOT hard" was made by merging sub-codes of maths as "not difficult", "maths as not hard", "maths as not tough" etc. These decisions were taken alongside readings of the quotations, so that decisions about code collapse took account of the qualitative data, whist also maximising frequencies for the collapsed, new potential cross case comparator codes.

The final stage in data preparation within ATLAS.ti was to create a number of supercodes, which made the data matrix that was exported into SPSS. The supercoding procedure was used to delimit the data to students' talk about their present course or present aspirations, so that we could restrict their data profiles to current values and learning approaches as applied to their mathematics programme. For example, if a student was coded as saying that they found maths hard then our system could locate the context in which that statement was made, for instance the context of their GCSE mathematics course, or as was required by this analysis to the context of their current AS maths or "Use of Maths" course. This was aided by the earlier addition of context and time codes to the data set, so that Boolean operators could be used to select the required data:

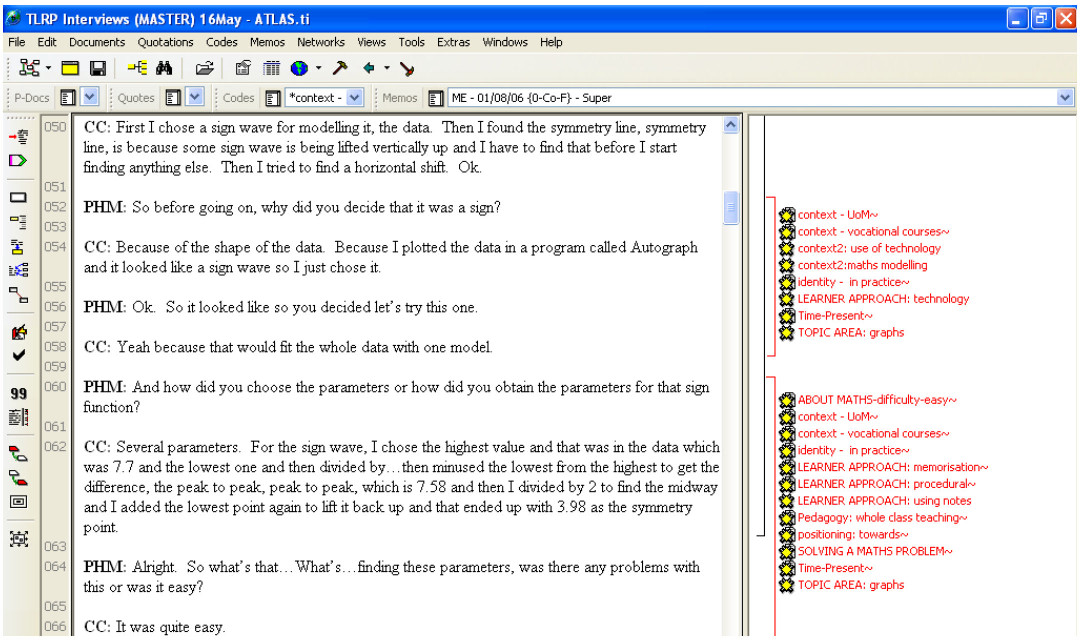


Figure 2 shows the axial coding system. This particular system is based on Flick (2002) so that codes can be read as a shorthand in place of the text (unlike the example provided in Figure One).

Once the data had been exported into SPSS, a systematic process of validation with the interview texts was completed to ensure the validity of the transferred codes, and changes were made as necessary, thus providing for additional validation checks on the coding system. The SPSS data set was embellished with a range of background variables, e.g. gender, Programme, college. This served the equivalent purpose to that of the “families” facility in ATLAS.ti

We had created the following comparator sets (see Table 1):

Table 1: Coded sets used in cluster analysis:

Value/Use Set (For Data Points 1,2 and 3[1]).

- *Maths is seen as useful for student's imagined career.*
- *Maths is not useful for student's imagined career.*
- *Maths is seen as useful for learning other subjects.*
- *Maths is not useful apart from when in class.*
- *Maths is useful in everyday life.*
- *Maths is not useful in everyday life.*
- *Maths has a perceived exchange value for the student.*
- *Maths as intrinsic value (this is not about a tool, its about them saying things like I love doing maths(application may or may not be important).Its about having a loive of the subject- intrinsic value for the person.*
- *Maths is hard*
- *Maths is not hard*
- *Maths is seen as useful for student's imagined career.*
- *Maths is not useful for student's imagined career.*
- *Maths is seen as useful for learning other subjects.*
- *Maths is not useful apart from when in class.*
- *Maths is useful in everyday life.*
- *Maths is not useful in everyday life.*
- *Maths has a perceived exchange value for the student.*
- *Maths as intrinsic value (e.g. indicative of having a love of the subject)*
- *Maths is hard*
- *Maths is not hard*
- *Maths is enjoyable*
- *Maths is not enjoyable*

Learner Approach Set (For Data Point3).

- *Student shows a conceptual approach to understanding maths at times.*
- *Student shows a procedural approach to understanding maths at times.*
- *Memorisation*
- *Maths as a Tool*
- *Maths as sole object of learning*

[1] Cluster analysis is reported only for one datapoint in this paper. Another analysis was conducted that examined change in cluster membership over time, but reportage of this is beyond the scope of this paper.

This is obviously a process that is reductive of the data. This was why it was important that care was taken with regard to the validity of the reductions made at each stage so that the resultant system had produced a simplified story for each student - a model.

The final stage in this process was to test the independence of the code-variables, which is a condition for application of The Two Step Cluster Analysis procedure. This can be differentiated from traditional clustering techniques, in its capacity to handle categorical variables. By assuming variables to be independent, a joint multinomial-normal distribution can be placed on categorical and continuous variables. The correlation matrix shows the correlation between the variables. In each cluster set all but one of the correlations is below 0.6 and most are reasonably close to zero, well under the 0.6 cut off that is usually used as a sign of collinearity. There is collinearity indicated for “maths as NOT useful for a career” and “maths as NOT useful in other subjects”, and collinearity between “learning approach procedural” and “learning approach: maths as object oriented, Not as a tool for solving problems”, and in both these correlations, r was approximately 0.7. Given that overall correlation between the 17 comparators was low, we went a head with the Two Stage Cluster Analysis on this basis. We note that methods for handing correlation of binary data are as yet under-development. For this reason both Pearson and Kendall correlation matrix statistics are provided in the Annex, though we are aware that more consideration of these data is needed.

We particularly point out that opposite sounding codes were not forced: that is, for any pairs of codes, e.g. “maths as useful in other subjects” and “maths as not useful in other subjects” an interview may be coded as (0,0), (1,0), (0, 1) or (1,1). This fits with the data, in that sometimes students do not identify with a certain comparator duality and sometimes they identity both, within a given interview, although, it was relatively rarely that (1,1) occurred, and also explains how we have constructed data sets which do not exhibit collinearity as a consequence of including such dual coding sub-sets. Coding with additional categories was explored, but the binary structure was needed, since adding additional categories meant that this size of dataset did not have the power to provide statistically significant comparators in the system; in addition, a binary structure retains meaning in that either the discourse was there or it was not, whereas it simply could not be assumed that the qualitative data should fit well into another classification.

In summary a number of phases were involved in preparing the data for Two Step Cluster analysis:

1 Open coding in ATLAS.ti,
2 Thematic analysis to identifying potential cross-case comparators,
3. Decisions as to which axial comparators were meaningful across the dataset,
4. Axial coding for these comparators with at least binary level of measurement,
5. Transfer into SPSS,
6. Validity checking using the interview transcripts to establish the credibility of the SPSS dataset,
7. Experimenting with comparator sets,
8. Establishing the independence of comparators in these sets.

Each of these stages involved making many judgements, and the Open Coding stage, especially, can be thought of as a process of start-up and exploration, until exhaustion.

Establish empirical categories of ways of identifying/participating with mathematics

A cross-interview cluster analysis, using the two step cluster procedure in SPSS[2], allowed us to explore the generality of categories of student subjectivity across the interview cohort and revealed a new layer of evidence for the importance of our ‘theory of mathematics value’ (Williams, 2008) and of ‘deep/surface’ learning approaches (Marton and Booth, 1997). This allowed us to explore ‘clusters of students’ connected by axial-code-sets for the sets “value/use of learning maths”, and “approaches to learning maths”.

Cluster Analysis: “Value exchange /use of learning maths” or “how maths fits into student life”

The analysis of the “value/use of maths” set signalled two, perhaps three clear differences in orientation towards the value of maths by students: ‘exchange oriented’, and ‘use oriented’, and also perhaps a ‘mixed’ group. The most strongly ‘exchange value oriented’ group of students were characterised as choosing maths for its currency as an entre to a successful career or university subject: some even disliked mathematics while seeing it as ‘good for the c.v.’, as a way of gaining an edge over competitors. “Using mathematics oriented” students tended to consider using mathematics as essential for achieving their immediate and longer term goals, e.g. understanding a physics problem or as essential knowledge for their intended future degree.

Two Stage Cluster analysis of “value/use of learning maths” codeset for DP1[3].

The model was first run with the available comparators. Four clusters were produced. However, two of these were significant only on one variable – maths is enjoyable. Subsequently, we took out *Maths is enjoyable* and *Maths is not enjoyable,* the result was a three cluster solution each with more than one significant contributing comparator. The three cluster solution had collapsed two of the clusters in the four cluster solution (there was very little other movement between clusters). This gives some credence to the acceptance of the three cluster solution. In addition with the three cluster solution each cluster had a number of statistically significant comparators.

[3] Similar clusterings were found for DPs 2 and DP3. The number of students at these data points was less and fewer comparators were significant, however, the fact that we found the same appearing clusters at all data points, gives more credence to the results of this analysis.

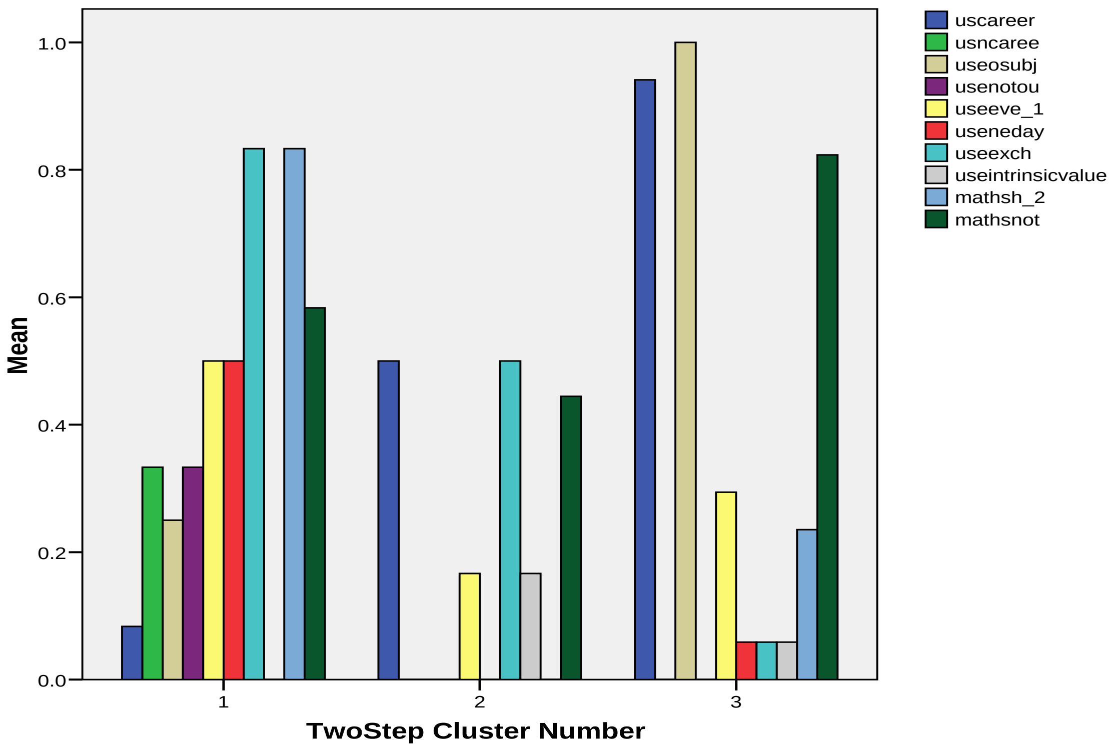


Figure 3: Three Cluster Solution for "value-exchange/use" comparator set.

Key for Figure 3:

Uscareer	*Maths is seen as useful for student's imagined career.*
Usncareer	*Maths is not useful for student's imagined career.*
Useosubj	*Maths is seen as useful for learning other subjects.*
Mathsnotou	*Maths is not useful apart from when in class.*
Useve_1	*Maths is useful in everyday life.*
Useneday	*Maths is not useful in everyday life.*
Useexch	*Maths has a perceived exchange value for the student.*
Useintrinsicvalue	*Maths as intrinsic value (this is not about a tool, it's about them saying things like I love doing maths (application may or may not be important).Its about having a love of the subject- intrinsic value for the person.*

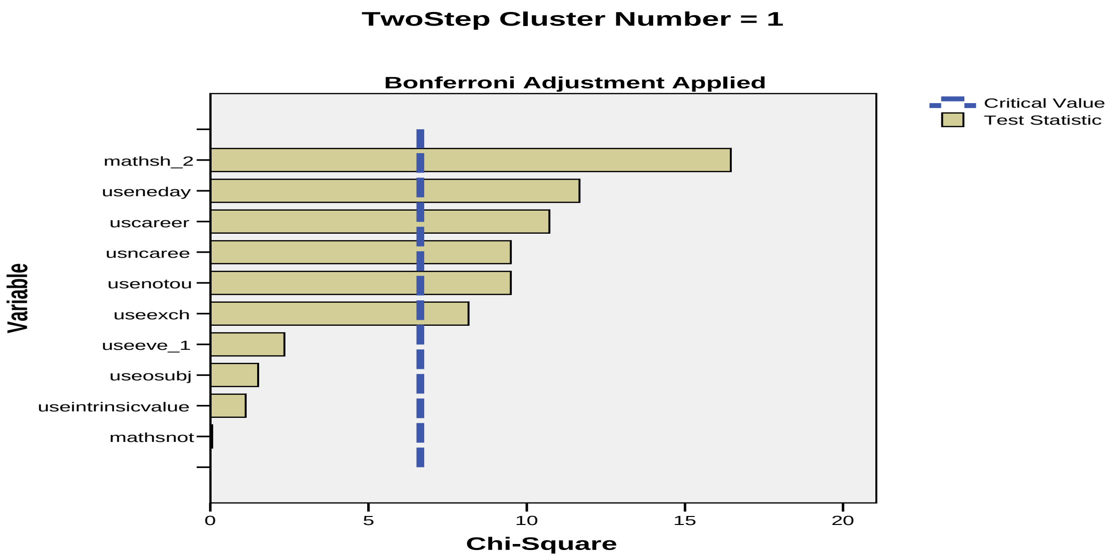
Mathsh_2	*Maths is hard.*
Mathsnot	*Maths is not hard*

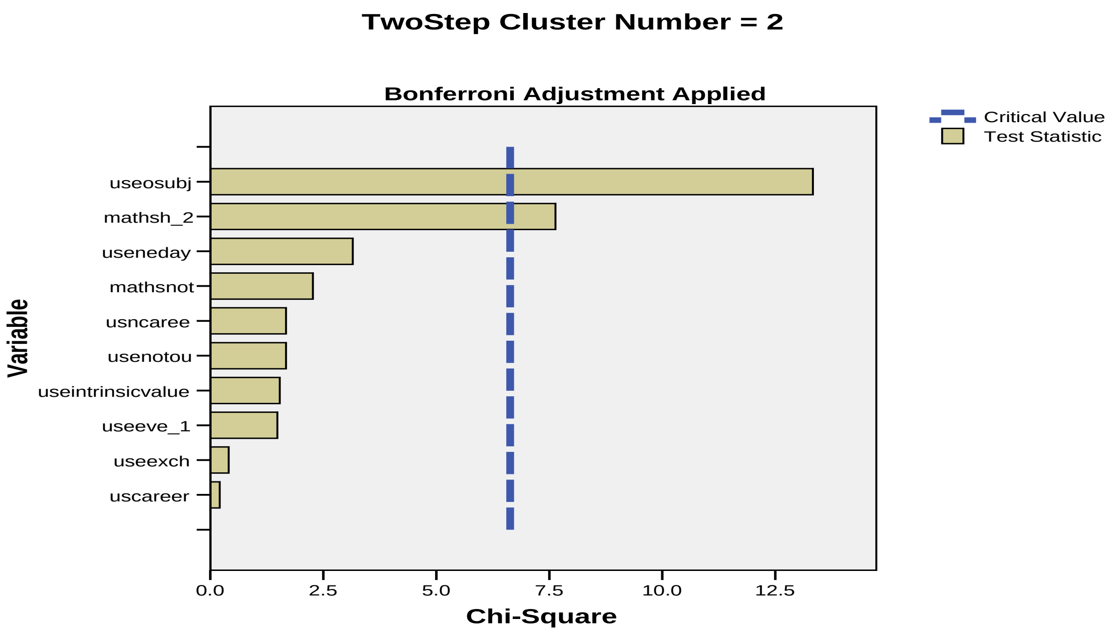
Cluster 1 Exchange orientation – Typically, the exchange value of mathematics dominates for this group, even though it is not seen as relevant to careers or being of use when learning other subjects. The relevance of mathematics to the everyday is acknowledged by some (though interviews suggest limitation to use of mathematics when shopping). Some students take mathematics even though or because it is hard, because they believe this makes them look good.

Cluster 2 Mixed orientation: Useful in future and is not hard - This cluster did not articulate the value of mathematics in their lives as much as those in the other two clusters. However, these students tended to talk about maths in terms of its exchange and in terms of its use. This may be best described as a NOT Cluster one or two category.

Cluster 3 Use orientation – Typically, students articulate an appreciation of the relevance of maths for desired career and of its immediate use in learning other subjects. There is little emphasis in students' talk on delayed gratification, even though many of these students stand to gain from having credentials in mathematics. Most students in this category describe mathematics as not being hard for them.

Figure 4 provides the attribute importance charts for this cluster analysis and also indicates which of the comparators were statistically significant. Figure 4 shows the cluster groups to be based on several statistically significant (95%) comparators. The "Attribute" Importance charts displays the comparators in descending importance. The dashed vertical lines mark the critical values for determining the significance of each comparator (variable). For a comparator to be considered significant, its "bar" must exceed the dashed line. For example, since the importance measures for the first six comparators exceed the critical value in the chart for cluster 1, we can conclude that all of the first six comparators contribute to the formation of the first cluster. The attribute importance charts, therefore, inform the interpretation of the cluster (Norusis, 2004). It follows that a "good" cluster will have multiple significant comparators, as otherwise we are as well using the single variable to divide the cases. Thus, cluster analysis allows for consideration of engagement in mathematics as multi-faceted and potentially connected articulations. Please note that significance is not an indictor of direction, a comparator may be significant in contributing to the formation of a cluster because it is mainly present (coded as 1), or because it is mainly not present (coded as 0).


TwoStep Cluster Number = 1
Bonferroni Adjustment Applied
Critical Value
Test Statistic
Variable
mathsh_2
useneday
uscareer
usncaree
usenotou
useexch
useeve_1
useosubj
useintrinsicvalue
mathsnot
0
5
10
15
20
Chi-Square


TwoStep Cluster Number = 2
Bonferroni Adjustment Applied
Critical Value
Test Statistic
Variable
useosubj
mathsh_2
useneday
mathsnot
usncaree
usenotou
useintrinsicvalue
useeve_1
useexch
uscareer
0.0
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10.0
12.5
Chi-Square

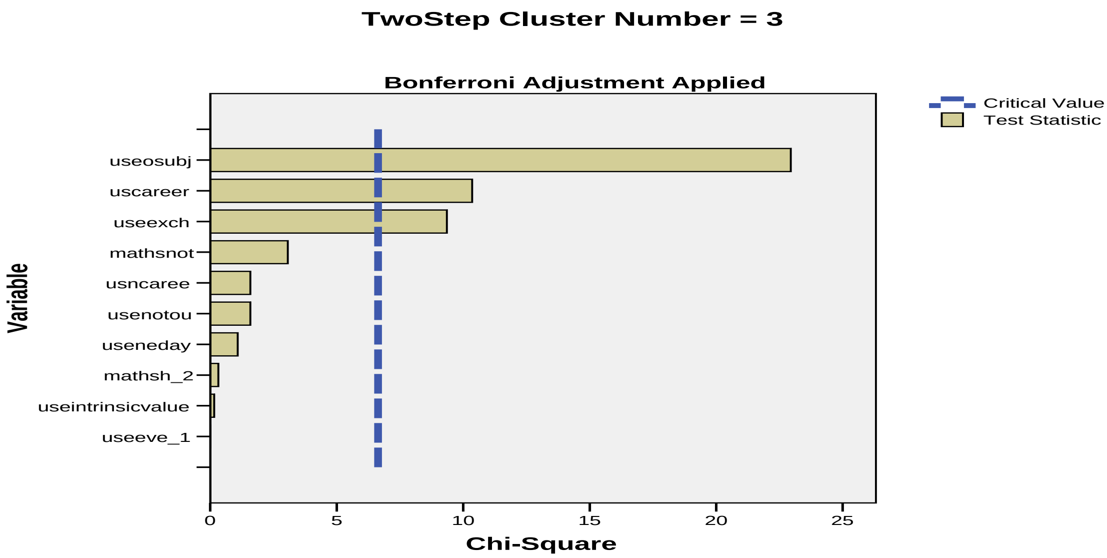
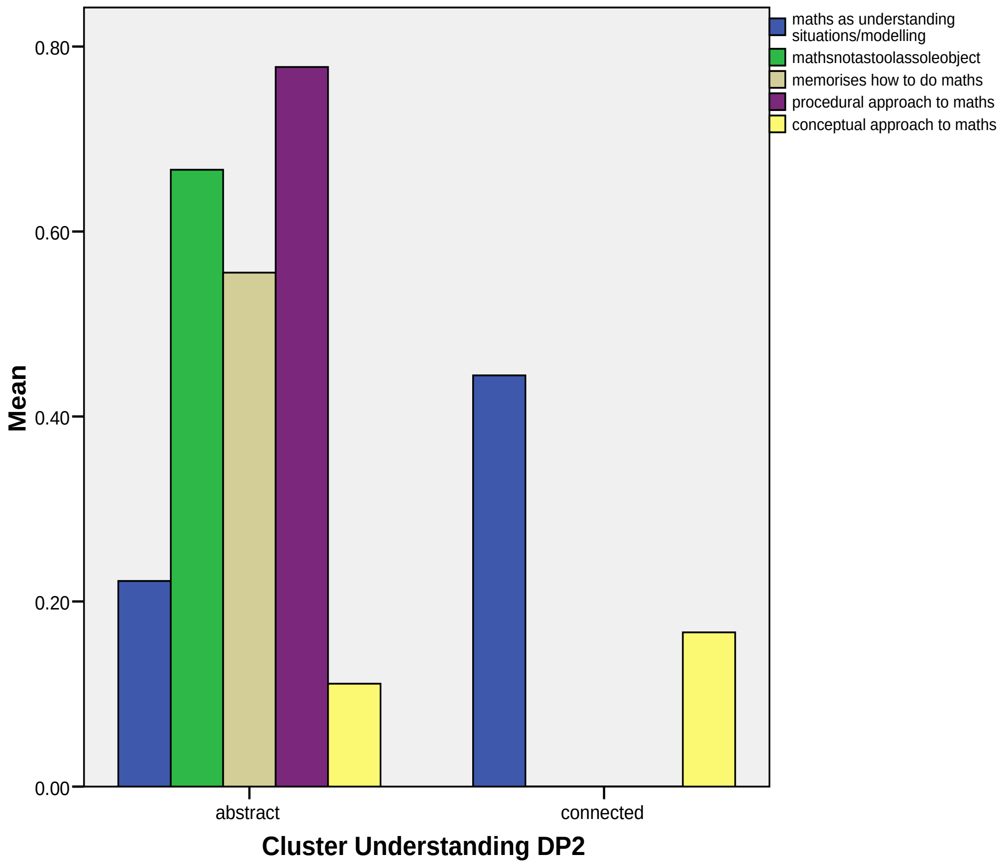


Figure 4 “Attribute” Important Charts for the “exchange/use” cluster group.

Cluster Analysis of “Approaches to learning mathematics”

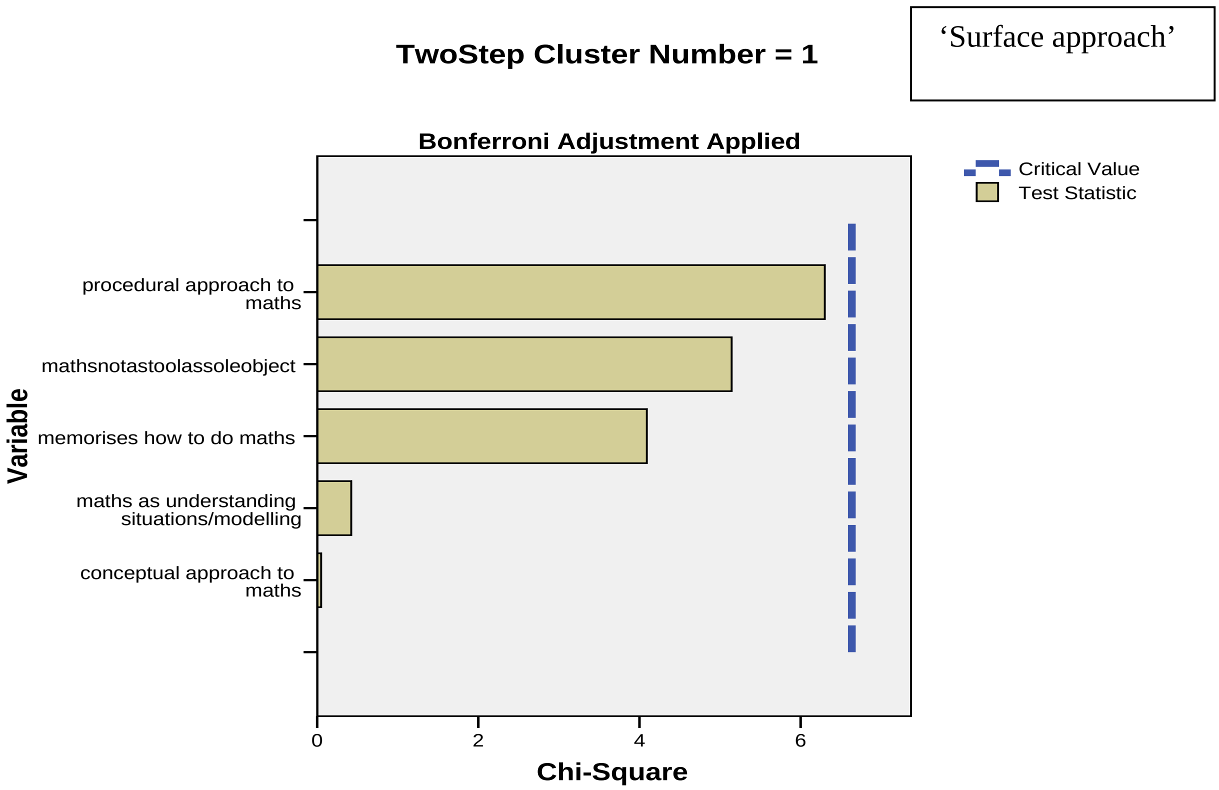
A similar cluster analysis was conducted for the “approaches to maths” set of codes. A significant cluster emerged around a construct of students who draw on procedural approaches to learning and see mathematics itself rather than applications as being the object of learning. This was indicative of a surface approach to learning as against a deep approach (Marton & Booth, 1997). A second cluster was identified that captures “all other approaches to learning maths”. However, there was **no** equivalent clear cluster around deep approaches to learning, though one can sometimes see in the interviews a deeper approach being adopted.

Figure 6: the clustering around ‘learner approach’ codes. NB the labels ‘abstract’ and ‘connected’ have been re-interpreted as “surface approach” and “NOT surface approach”

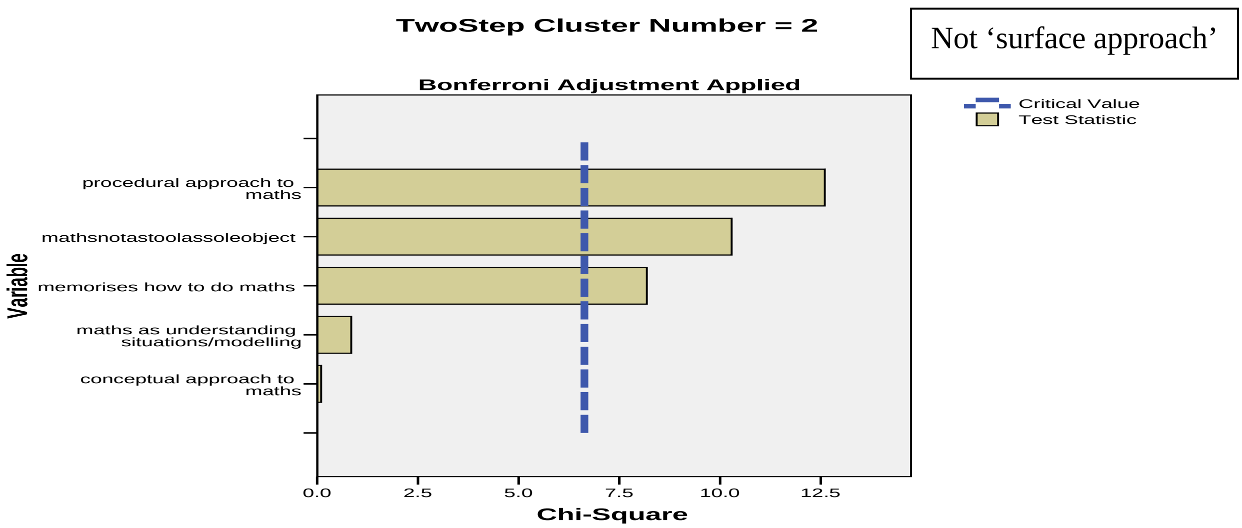


0.80
0.60
0.40
0.20
0.00
Mean
abstract
connected
Cluster Understanding DP2
maths as understanding situations/modelling
mathsnotastoolassoleobject
memorises how to do maths
procedural approach to maths
conceptual approach to maths

•



'Surface approach'
TwoStep Cluster Number = 1
Bonferroni Adjustment Applied
Critical Value
Test Statistic
Variable
procedural approach to maths
mathsnotastoolassoleobject
memorises how to do maths
maths as understanding situations/modelling
conceptual approach to maths
0
2
4
6
Chi-Square



Not 'surface approach'
TwoStep Cluster Number = 2
Bonferroni Adjustment Applied
Critical Value
Test Statistic
Variable
procedural approach to maths
mathsnotastoolassoleobject
memorises how to do maths
maths as understanding situations/modelling
conceptual approach to maths
0.0
2.5
5.0
7.5
10.0
12.5
Chi-Square

The “attribute” importance charts in Figure 6a,b,c show that the “surface approach cluster” is based on three of the five statistically significant comparators: mathematics as procedural, maths not as a tool but as sole object of learning, and memorisation. It is noted, however, that the first two were related r=0.7 in the original data set; however, even if one of these were removed we still find the same cluster. The other cluster has an empty-set of statistically significant comparators, and for this reason we interpret it as “NOT” the other cluster.

The meaning of the clusters was then established by considering the attribute importance charts and the cluster bar charts. In addition, we would expect, if the cluster groupings were meaningful, that the narratives (interviews) of students in a given cluster should be clearly seen to articulate similar values, and that there should be distinct differences between clusterings. We therefore checked this by considering which students had been placed in each cluster, and based on our knowledge of the interview data and some additional spot check readings were able to conclude that the clusters were meaningful, though we believe that their real validity rests on their capacity in practice to provide for analysis.

Students/interviews can be so classified in terms of cluster membership. If we consider identification with maths as a simple model of membership to the clusters in the two analyses described in this paper then the interview sample can be classified as “exchange oriented”& “surface approach”, “exchange oriented”& “NOT surface approach”, “use”& “surface approach” and “use oriented”& “NOT surface approach”.

Two strikingly different ways of participating in mathematics can be seen between students in the “exchange oriented”& “surface approach”, “use oriented”& “NOT surface approach”. These students’ discourse can be considered with regard to how they align themselves with maths. We suggest that in the former case (e.g. Joseph) alignment or identification with maths was seen as somewhat disconnected but that in the later case (e.g. Vladimir) a more intimate connection with mathematics as useful was demonstrated. Thus, we can view the clusters as picking up on kinds of identification with mathematics. If membership of exchange/surface and use/NOT surface is taken to demark extremes in students ways of identifying with mathematics, then we might see other membership combinations as presenting identification with mathematics that is somewhere in between.

The relation between Programme and ways of identifying/participating with mathematics.

Table 1: Cross-tabulation for cluster membership at DP2

Cross tabulation for cluster membership For DP2	Use Oriented	Exchange Oriented	Mixed
Surface approach	1	3 All AS Maths GE	5
Not surface approach	9 e.g Vladimir BTEC Engineering Use of Mathematics	6	2 e.g. Craig Use of Mathematics GE

Table 1 shows that a use of mathematics orientation, (with regard to the value of mathematics in student's lives) is associated with a NOT surface approach to learning (9/10 "use oriented" students were in the NOT surface approach group.

However, Table 1 does not account for the students' programme of learning. Figure 7 is an X-Chart of Cluster membership split by Programme. X-Charts allow patterns in the data to be detected with ease visually (if they are there!). The X-Chart is based on the data in Table One and has been panelled by Programme, so to display any differences in the cluster relations between Programmes.

When considering how Programme/pedagogy might influence students' engagement, the cluster analyses allow us to look at the distribution of cluster membership across programmes, and so detect differences in patterns of discourse (these being embedded in the clusters) in students' meta-pedagogic contexts. In Figure 7 we can see that all BTEC Engineering students are in the "non-surface approach" cluster and that the majority are also in the "use-oriented" cluster. On the other hand, the AS Level Maths students are weighted towards the surface and either "exchange oriented or "mixed orientation" clusters. Only one of the AS maths students was classified in the "use orientation cluster". There appears to be no clear pattern for the General Education AS "Use of mathematics "students, and it should be noted that these trends are based on small numbers.

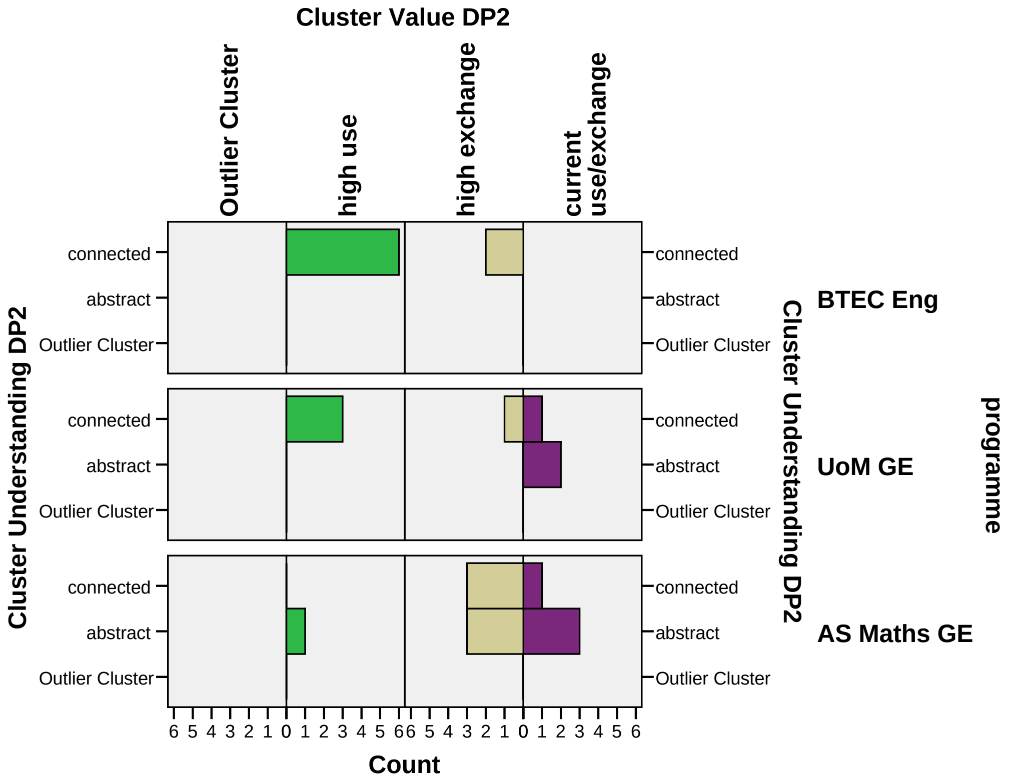


Figure 7: X-Chart of Cluster membership split by Programme of Learning.

The chart presented in Figure 7 does appear to be suggestive that certain kinds of engagement with learning mathematics is related to programme. Especially, there appears to be a difference in the ways in which General Education AS Mathematics students and BTEC engineering students taking AS Use of Mathematics tend to identify with mathematics. The 9:1 ratio of interest in Table 1 can, thus, be explained by Programme.

Discourses in Students' Meta-pedagogic contexts: an interpretation of Figure 1.

Our other case study work, including lesson observation and interviews with teachers, led us to identify distinct discourses of education to do with performativity - in the audit culture of the colleges, in the discourse of teachers about exams, in teacher talk in the classroom and in students' talk about the value of learning (see Williams et al, 2008). We also observed a discourse of "use" amongst some students, especially engineering students (e.g. see Hernandez-Martinez et al, 2008). The cluster analysis supports these claims, but also goes further because it establishes that (i) there are distinct discourses in the interviews to do with valuing mathematics and learning approach, that (ii) valuing maths for its use and learning approach are related, and that (iii) this relation occurred, especially in the context of the BTEC engineering Programme. Given that learning approach may be anticipated to be influence by pedagogic culture.

The AS “Use of Mathematics” BTEC Engineering students were learning in a pedagogic context in which they were, by and large, already personally committed to becoming engineers (those less committed appear in the “exchange –oriented” cluster, perhaps because these two students had decided on different pathways in mathematics and computer science, We speculate that some engineering students may have already seen themselves as beginning engineers, for example, “*When I actually bought an electronics kit when I was little and that actually got me into fixing stuff and I wanted to be an Engineer when I grew up that actually made me like Maths more. I wanted to do more Maths in the future.”* We consistently found positive relationships with mathematics expressed by these BTEC engineering students. They appeared as engaged in a discourse that consistently reinforced the necessity of mathematics, exemplified by Vladimir’s claim that engineering *is* mathematics (we found this to be a common belief), and they accept they will need mathematics in their future careers, so much so, that this appeared as part of their discourse. As a confirmation, this view was also reinforced by their teachers during interviews. To give another example, ‘*Maths is the main part of electronics. Because to work out what is wrong, you have to work out the formula. You need Maths to calculate stuff,* to *calculate voltage to current. You need lots of Maths to do that as well. So my Maths course right now is helping me to do my electronics course as well. That’s why if I don’t do Maths I can’t do electronics.’* In the above extract the AS Use of Mathematics course is viewed as integrated with engineering, so that the mathematics they do as part of the mathematics course is set within engineering contexts, and they can then use the mathematics they have learnt to do engineering. The emphasis in the Use of Mathematics curriculum on conceptual understanding, through modelling and use of technology, is reflected in the kind of approaches to mathematics that they experience, for example Mahmood, earlier, when he talked about using logarithms in the reduction to a straight line in order to solve a problem.

. What is more the BTEC maths students often talked with confidence about their mathematical ability, often regarding mathematics as being their best subject when at school, mathematics seemed to be a subject that these boys identified with, and we gained a sense that they believed it was Ok to like mathematics. For example, *‘I’m not brilliant, I am just good. Well hopefully, as Maths is quite important I will hopefully get better by the end of the year and understand more, it is going to be a big part because it is almost in everything, electronics as well.’* Or alternatively, “*Yeah, I like maths. Because I’m doing electronics course and maths is very closely related to the engineering and the drawings and yeah, I think that’s…if you’re good at maths, you will be very successful in engineering so yeah. I enjoy maths. I think it’s very important*.” An other example, again shows a positive identification with mathematics, typical of the BTEC engineers,

PHM: What makes you be in there?
Punab:I think maths in something I can do, something I can understand easily. It’s not something like English, which is one of my worst subjects where it’s like words and literature - I cannot do that. But Maths, where it’s more like logic, something like that, where I can put in one thing and something will come out, something like that, I am better with.

In the context of engineering there was a strong discourse of **use of mathematics**, and about themselves as **users** of mathematics that seemingly received little in the way of opposition within the context of their full-time BTEC Engineering Programme. It is this discourse that we believe the "use-oriented with not-surface approach" clusterings detect.

When we looked to the interview data we could confirm the relationship picked up by the cross-tablulation of cluster membership in that the BTEC engineers tended to talk about using maths and learning maths in related ways, whereas, usually this wasn't apparent in the talk of other students, the exception being a small sub-set of Use of Mathematics General Education students, who were also located in the "use-oriented" with "non-surface" approach clusters. Two of these wanted to be a mechanical engineer and one transferred to engineering A2 in her 2nd year of 6th form college.

Manjit (Use of Mathematics GE – "use orientation" with "non-surface" cluster), for example, talks about "*the maths behind it*" (a conceptual approach) to using mathematics, "*In the designs and all the product making and stuff, you have to learn about different formulas and different techniques of building. Finding areas, dimensions and stuff like that. It's really quite fun to actually see the maths behind it, it's kind of like physics in a way, which I like."*

Alternatively, Vladimir (Use of Mathematics BTEC engineer "use orientation" with "non-surface" cluster**)** demonstrates conceptual understanding in relation to modelling a problem in engineering. Here we see Vladimir taking almost interchangeably about mathematics and engineering. *Because we were given some functions, linear, quadratic, exponential, trigonometric, and you had to choose so just explain a little bit about the exponential. Why it is not possible to use exponential for this type of data*.

PHM: *Why…what were your reasons for not choosing an exponential? For example, here you said you used a linear one. Why wouldn't you use an exponential one? Why?*

V: *Because it won't fit the data, basically. I just realised that the linear function is the best function to model this particular data and trigonometric for the whole data.*

PHM: *What's the criteria for deciding if it fits or not? How do you know if it fits or not?*

V: *Basically, I know that the trigonometric functions…it's a periodic function so basically the voltage that changes over time, it's a periodic change. It goes from minimum to maximum value and then goes back to minimum and then back to maximum. It's periodic and the only function to know is the trigonometric function. Yes.*

PHM: *And for example, you said that you used quadratic to model this part of the problem and then you used the linear to model just this part of the problem. So how do you decide which of the models is best? Let's say we take just this part. How do you decide either the quadratic or the linear is the best model?*

V: *Yeah. I found it using graphs, just to show how they compare. Here is the quadratic function. So I've got the trigonometric graph and the quadratic graph and the linear graph so I* *can compare the three of them together so this is my trigonometric graph and this my quadratic graph. We can see which one is the best, obvious for this particular part of the data.*

PHM: *Ok. So here, is it a…just because the points are closer to the function.*

V: *In these points? Because there is an error here. Because the voltage changes with time, it's from a railroad station. It's not a perfect model. That's why my model doesn't pass through every point, basically.*

We have many similar examples that provide illustrations.

Of the seven AS "Use of Mathematics" students on a General Education Programme only one (Jean earlier) was classified in the "surface approach" category, three were in the "use with non-surface approach" category, and the remaining three were in other groups. Although there was no significant relationship found between the value and learning approach cluster categories for this group, if we go back to the interviews we can see a that this group of students talk in various ways about mathematics, including some drawing on a discourse more similar to the engineers. There was, however, no engineering mathematics discourse apparent amongst the AS Mathematics General Education students.

We suggest that Use of Mathematics students were sometimes in dialogue (Bakhtin, 1981) with a curriculum/programme narrative, which emphasises an appreciation of mathematical uses in society, "applying mathematics in situations" and conceptual understanding through modelling. All the Use of Mathematics students were coded positive on the single comparator code mathematics as useful in the everyday, and we can see how students' drew on this narrative in different ways e.g. Jean earlier ("surface approach" with "mixed value" cluster) gave both the course narrative view and her own, indicating that she hadn't fully internalised this part of the new curriculum narrative. On the other hand, Craig ("not surface", with "mixed valued" cluster), for example, draws on this narrative when describing mathematics as being about application in various situations.

C: *Well, maths as I see **normal**, is you learn something and you just do it, you know. You just learn it and you apply it to a question. Whereas in "Use of Maths" you use it and you apply it to a certain situation rather than just a question, if you know what I mean.*
MP: *Yeah.*
C**:** *So it was quite a different way of learning and quite a different way of teaching, really, but once you got used to it it's quite…I think it helps. …Well, I guess it's just the way it's done. .…because when you're taught it you're not just learning it, are you? You're learning about how it's applied to a certain situation and not just how to do it because I think, like I say, normal maths is just learning how to do it and how to do this. Like, for instance, multiplication. You learn how to do it and you apply it to a simple question. You don't look deeper into it and look at how you could use it in real life which is what use of maths is all about, really, isn't it?*

We also see how Craig draws on this narrative of use in his practice of mathematics, which we can see in the frequent references made to the speed of a vehicle in relation to the graph, rather than to an algebraic expression using abstractions e.g. x's and y's when talking through his solving of the problem. "*It's gone steeper and closer to the line so it's actually going bigger, it's going faster than before so…but if you make it lower, things are going to go…you can change it again. It's at more of an angle so you can see the B is actually changing the speed of the vehicle, I guess. Because it's changing the steepness of the graph and that's what's just changed. I changed it from, I've changed B from 5 to one and seeing the effects of that and you can tell by that what it's doing to the graph and you can interpret it by thinking about how it relates to the data that you're using on the graph. So for instance, If something's going from that to that, it's going at less speed, isn't it? It's at more of an angle so you can see B is actually changing the speed of the vehicle, I guess.* This reflection appears to verify a self- awareness that he is making an interpretation of a problem using graphics, '*And you can interpret that sort of thing from the graphics. That's what I did there.* This is perhaps evidence of internalisation of the course narrative taking place, and seems to bring his talk about doing mathematics nearer to the discourse of mathematical modeling used more typically by the engineers.

A**ll the Use of Mathematics students** shared an awareness that there are more diverse ways of doing maths, even if this appreciation for use did not necessarily spill over into their practice, e.g. Suzanne talked about Use of Mathematics as "*not normal maths*" and involving using graphs to understand situations but then ignored the contextual information provided and chose to rely on procedural algebraic approaches when talking aloud to us about solving a problem, which could have been tackled in different ways, so:

Suzanne: *Oh, you…that one's just solving the equation. Solving 0.5 equals e to the minus blah blah blah and you have to find t by taking ln [natural logarithms] on both sides I think.*
PD: *And is that how you did it? You kind of did it using…*
S: *Like, here.*
PD: *ln, you said.*
S: *ln*
PD*: So did you do that more as a handwritten calculation?*
S: *Yeah.*
PD: *Solving the equation* [algebraic].
S: *This one. I think it's that.*
PD*: Right. Let's see. Then show it takes approximately 5730 years. Yeah. Is that comforting? When you find that you've got the answer?*

Conclusion and discussion

We claim that the cluster analysis establishes systematically on the basis of the entire interview set at Data Point 3 that:

(i) there are significant, distinct patterns of discourses in the interviews to do with 'valuing mathematics' and 'learning approach' amongst the students,
(ii) valuing maths for its use and learning approach are significantly related, and
(iii) this relation can sometimes be explained by Programme, especially in the context of the BTEC engineering Programme which students followed concurrently with "Uses of mathematics"..

We interpret this relation between Programme, values and learner approach as an interweaving of a discourse of ‘being a mathematics user’ or ‘being a mathematical modeller’ that some students were acquiring. It seems that this way of being might be a result of engaging in dialogue with a Use of Mathematics curriculum narrative. Conceptually, we suggest after Bakhtin (1981) that students in dialogue with this Programme narrative find a space in which to discursively construct themselves as a different kind of mathematics learner – a mathematical modeller.

We speculate (data from classroom observations and texts were suggestive of this) that a pedagogic/curriculum Use of Mathematics narrative was narrated within a classroom activity system, whereby mathematics is sometimes used as a tool for problem solving through mathematical modelling. We also suggest, that the demands within the assessment of the Use of Mathematics course for students to provide evidence of conceptual understanding in course work makes available to them opportunities to become users of mathematics through modelling (see Hernandez-Martinez et al, 2008), and that this aspect of the curriculum is important in influencing how students view mathematics and sometimes how they approach their problem solving.

We suggest then that there exists a discourse of use of mathematics and of being/becoming users of mathematics which aligns with and is developed by engagement with a pedagogic/curriculum Use of Mathematics course narrative. This seems particularly well aligned with a discourse of future use and expectancy for mathematics to have a place in their imagined futures, which was drawn on especially by the engineers, and reinforced to them in the discourse of their full-time BTEC programmes. A dialogue between the students’ sense of identification with their imagined futures and the course narrative may take place for some students, and we speculate that a dialogue may be especially working for the engineers because they tended to have particularly strong designated identities (Sfard & Prusak, 2005) with which learning mathematics was an aspect, “*Yeah, I like maths. Because I’m doing electronics course and maths is very closely related to the engineering and the drawings and yeah, I think that’s…if you’re good at maths, you will be very successful in engineering so yeah. I enjoy maths. I think it’s very important*.” We hypothesis that for students who were not intending to continue on with mathematics and use maths in the future, that there was less space for an internal dialogue to resonate. The Engineers were atypical in this respect, because about half students taking mathematics were not intending on a mathematically demanding degree (Davis et al, 2008, working paper a).

By contrast, the small group of “Exchange-oriented” with “Surface approach” learners were general education students taking AS maths. These students talked about maths is ways that were suggestive of a discourse of disconnection from mathematics, (as for Joseph, Martin and Louise, earlier) although overall there was no relationship between value and learning approach cluster membership. However, this does not imply that there *is* no relationship between the value of mathematics as exchange and learning approach as surface, but rather simply that we haven’t evidence of positive confirmation of such a relationship based on this analysis, and speculate that this may be a consequence of small numbers.

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