MEASURING PERCEIVED SELF-EFFICACY IN APPLYING MATHEMATICS

Wake G.D., Pampaka M.

University of Manchester, U.K.

In England a new course of post-compulsory study has been developed based on the premise that developing confidence and ability in applying and modelling with mathematics will better prepare students to elect to study courses that require relatively high levels of mathematics in their future studies in Further and Higher Education. In investigating this we have designed an instrument to measure perceived self-efficacy beliefs in applying mathematics. Here we report principles of construction of the instrument together with initial analysis which suggests that it does allow measure of perceived self-efficacy beliefs in mathematics generally and in pure and applied mathematics separately with early evidence suggesting that the new course is successfully developing students' confidence in applying mathematics.

INTRODUCTION AND BACKGROUND

The "mathematics problem" in England is deep seated: too few students are well prepared to continue their studies from schools and colleges into courses in Higher Education Institutions (HEIs) in mathematically demanding subjects. Concerns have been raised by those involved in the sector and this is reflected in national reports such as the Roberts Review (2002) which focussed on the supply of well qualified graduates in science, engineering and technology and the Smith Inquiry (2004) which investigated mathematics in the 14-19 curriculum.

Within this contextual background this paper reports one aspect of the work of an ESRC (Economic and Social Research Council) research project, 'Keeping open the door to mathematically demanding courses in Further and Higher Education'. This mixed methods project involves both case study research investigating classroom cultures and pedagogic practices and individual students' narratives of identity together with quantitative analysis of measures of value added to learning outcomes in an attempt to investigate the effectiveness of two different programmes of study. Here we report the development of an instrument designed, as part of this latter strand of the project, to measure perceived self-efficacy in applying mathematics.

Central to our research is a new qualification in the post-compulsory (post-16) sector that was specifically designed to better prepare students to continue with study of mathematically demanding subjects. This recognises that the target students will not wish to study mathematics for its own sake: high achieving students, aged 16-19, wishing to study mathematics, engineering or physical science courses at university will follow non-compulsory courses leading to the "traditional" mathematics qualification – A Level mathematics. It is the intention that the new qualification will be accessible to students who are likely to be starting from a lower base of prior attainment in mathematics, but who nonetheless wish to go on to study a course that makes considerable mathematical demands at university. This qualification, AS Use of Mathematics, is designed as equivalent to study of the first half of an A Level in the English system (although a full A Level "Use of Mathematics" is not currently available), and includes considerable preparation in the use of algebra, functions and graphs, with an option to study either "modelling with calculus" or "using and applying statistics"[1]. Due to its potential students, the new qualification, therefore, attempts to ensure that those who study it will see an immediate or potential value in mathematics within their experience of other study, work or interests. Therefore, mathematical modelling and applications are fundamental to courses leading to the AS "Use of Mathematics". Whilst students may not be explicitly taught "to model", the philosophy is such that the mathematics, as learned and practised, involves being actively engaged in aspects of mathematical modelling and making sense of commonly used mathematical models with a particular emphasis on critical awareness of how modelling assumptions affect the validity of solutions to problems.

This course contrasts with the standard or "traditional" AS / A Level route which is followed by the majority of 16-19 year-old students in preparation for further study in HEIs. In this case applications and modelling play a much less prominent role: the emphasis is on a high level of technical facility with subject content in "core" or "pure" mathematics.

Our concern, then, is to investigate practices that widen participation in the study of mathematics: consequently in evaluating the effectiveness of the different programmes, AS Mathematics (the first half of study towards a full A Level in mathematics and referred to here as AS Trad) and AS Use of Mathematics (AS UoM), one measure we are investigating is perceived self-efficacy in mathematics and in particular perceived self-efficacy in applying mathematics.

PERCEIVED SELF-EFFICACY

It is now almost thirty years since Bandura (1977) proposed the construct of perceived self-efficacy: "beliefs about one's own ability to successfully perform a given behaviour". He later situated this within a social cognitive theory of human behaviour (Bandura, 1986), before more recently developing this further within a theory of personal and collective agency (Bandura, 1997).

Perceived self-efficacy beliefs have been explored in a wide range of disciplines and settings including educational research where they have been investigated in relation to progression to further study and career choices (Lent and Hackett, 1987) and in relation to affective and motivational domains and their influence on students' performance and achievement. One's perceived self-efficacy expectations are

[1] The specifications and assessment associated with the new qualifications were designed by one of the authors (Wake) acting as consultant to the government's Qualifications and Curriculum Authority.

accepted to play a crucial role in determining one's behaviour with regard to how much effort one will expend on a given task and for how long this will be maintained. Behaviour therefore is crucially mediated by beliefs about one's capabilities. Bearing in mind that outcomes which individuals consider as successful will raise perceived self-efficacy and those that they consider as unsuccessful will lower it, we hypothesise that those students following a "Use of Maths" course will increase their perceived self-efficacy in relation to applying mathematics in particular, to mathematics in general, and this will have a positive effect on their likelihood of further study that requires mathematics. This may be particularly important in widening participation into mathematically demanding courses in Higher Education as the AS UoM course at present is often catering for those on the margins of studying mathematics in Further Education College courses.

Perhaps most important and relevant to our study are research findings that suggest that perceived self-efficacy in mathematics is more predictive of students' choices of mathematically related courses in programmes of further study than prior attainment or outcome expectations (see for example, Hackett & Betz, 1989 and Pajares & Miller, 1994). Hence, our project's need for an instrument to measure perceived self-efficacy in applying mathematics. Here we describe the underlying framework which we used in development of this instrument with particular reference to constructs relating to mathematical modelling (Blum, 2002) and "general mathematical competences" (Williams et al., 1999). Whilst drawing on these constructs from mathematics education we have also taken into account, as we shall illustrate, the important advice Bandura offers those building measures of perceived self-efficacy; namely, that they need to be clear in specificity of the tasks that respondents are asked to judge, paying particular attention to levels of task demand, strength of belief, and generality of the task.

CONCEPTUALISING APPLICATIONS OF, AND MODELLING WITH MATHEMATICS

For many years the work of the ICTMA (International Conference for the Teaching of Mathematics and its Applications) group has explored how mathematical modelling can inform teaching and learning of mathematics at all levels. An important result of the work of members of this group is the conceptualisation of mathematical modelling and how this relates to applications of mathematics. Whilst there is not room here to discuss this fully we would draw attention to some of the main features of mathematical modelling, how this relates to "applications of mathematics", and how this is usually conceptualised in implementation of mathematics curricula in schools and colleges in England.

Essential to using mathematics to model a real world situation or problem is the genesis of the activity in the real world itself. Mathematising this situation, that is simplifying and structuring it so that it can be described and analysed using mathematical ideas and constructs, leads to the mathematical model. Following analysis using mathematical knowledge, skills, techniques and understanding the outcomes and results are interpreted in terms of the original problem, being checked to determine whether or not they are valid. At this stage it may be decided that the model is adequate, or that it needs to be modified in some way, perhaps making it more sophisticated so that the results/solution to the problem are more appropriate. This can therefore be conceived of as a cyclical process with the "modeller" translating between real world and mathematical representation. Some mathematical model types are commonly found and used to describe many different situations (for example, in the sciences models of direct proportion, exponential growth and decay and inverse square laws abound) and in some instances a recognition of this allows the modeller to short-circuit some of the process and work quickly between mathematical model and real world. In the discussion document which set out the agenda for the forthcoming ICMI study of Applications and Modelling in Mathematics Education (Blum, 2002), care was taken to distinguish between use of the term "modelling" on the one hand, to describe the mathematisation as one moves from reality to mathematical model, and "application" on the other as one interprets mathematical analysis in real terms, sometimes from a given mathematical model.

In recent years in England, scant attention has been paid to the process of mathematical modelling in "traditional" courses at this level with assessment encouraging a view of problem solving / applications being something that follows learning and, if possible mastery, of "basic" techniques. As has already been suggested, the new AS Use of Mathematics attempts to bring to the fore the processes of modelling and particularly application as outlined here.

THE SELF-EFFICACY INSTRUMENT

In developing an organising framework around which to build our self-efficacy instrument we turned to the construct of 'general mathematical competence' (for further discussion of this see for example, Williams, Wake and Jervis, 1999). This acknowledges that, as suggested above, in certain domains there are common ways of bringing mathematics together to solve problems and model situations. So mathematical modelling as practised in a range of situations by learners or workers (for example see Wake, 2007) is not a wholly open practice but is often based on common patterns of working that we have identified and briefly outline below.

In summary, a general mathematical competence is the ability to perform a synthesis of general mathematical skills across a range of situations, bringing together a coherent body of mathematical knowledge, skills and models with attention being paid to underlying assumptions and validity. Crucially then, the construct of *general mathematical competence* moves us away from thinking of mathematics as a collection of atomised mathematical skills towards consideration of it as a practice in which we bring together and use mathematics (often in common ways) to analyse situations and solve problems.

The general mathematical competences (g.m.c.s) developed were:

Costing a project: This requires the calculation of the 'cost', in monetary or other terms, of a (substantial) project or activity. Graphical / visual display of findings may be required.
Handling experimental data graphically: Developing a graphical display of experimental data requires firstly the identification of suitable data, its collection, organisation and recording prior to any scaling or processing that may be required. Following actual plotting of the raw or processed data identification and validation of mathematical functions to appropriately model the data may be necessary.
Interpreting large data sets: We increasingly have access to large sets of primary and secondary data. This g.m.c. requires initial sifting of data and identification of appropriate hypotheses, followed by the selection of the data required. Calculation of appropriate measures of location and spread and the development of appropriate visual / graphical display allow interpretation in terms of the original hypotheses.
Using mathematical diagrams: This g.m.c. requires the ability to translate reality into a diagrammatic representation (including plans or scale drawings) and vice versa.
Using models of direct proportion: The use of models of direct proportion permeates many areas of mathematical application (e.g. other g.m.c.s require the scaling of data which requires an understanding of the concept of proportionality). This g.m.c. develops numerical, graphical and algebraic understanding of this key mathematical concept requiring that one can move with ease between these different modes.
Using formulae: This g.m.c. pays attention to algebraic form and the use of algebraic formulae. It recognises that one often needs to be able to use formulae to process data and therefore requires that one is able to select appropriate data to use within any algebraic expression paying attention to units / dimensions of quantities.
Measuring: In practical work in science and technology it is important that attention is paid to measurement of data. In particular it is important that due attention is paid to the use to which the raw and processed data will be put as this will inform not only what should be measured but also the required accuracy and units with which quantities should be measured. Calibration of instruments is covered in this g.m.c.

A total of thirty items were developed with twenty four based on the seven general mathematical competences identified above and a further six being developed in "pure" mathematics areas. The items were designed at three different levels, it being the intention that, as part of the long term project, the instrument will be administered to the same student population at the start and end of a single academic year and at one point just beyond this. The different levels of items will allow for the increasing mathematical maturity of the cohort being studied. As usual in self efficacy studies, students were asked to choose the level of their confidence in solving each item (but, it was stressed, without the need for actually solving the item) using a Likert type scale. Examples of "pure" and "applied" items are given in Figure 1.

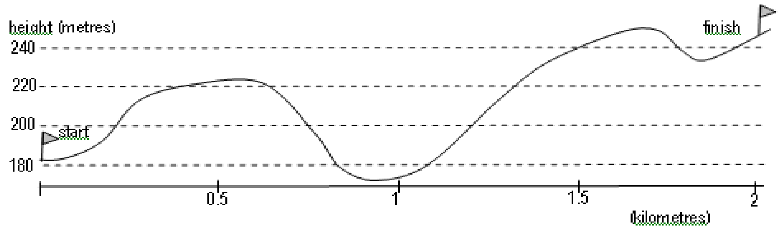
C16 Solving a linear and a quadratic equation simultaneously when there are two different solutions, such as:	not confident at all	Not very confident	fairly confident	Very confident
	1	2	3	4

Find the two points of intersection of the straight line, $y + x = 5$, with the parabola, $y = x^2 - 2x + 1$.

A/2

C6 Solving practical problems using charts, arithmetic and scales, such as:	not confident at all	Not very confident	fairly confident	Very confident
	1	2	3	4

The diagram below shows the height / distance profile of a walk in Derbyshire with height in metres and horizontal distance in kilometres.
Taking into account these different scales estimate as accurately as you can, using Pythagoras' Theorem, the distance walked **down** hill.



MD/2

Figure 1. Sample pure (top) and applied (bottom) items from self-efficacy instrument.

As these sample items illustrate, whilst they give a general description of the type of activity required, each ensures a greater degree of specificity by including an example of the type of task. Whilst item C16 requires application of mathematical understanding and techniques this is taken to be in the discipline of mathematics itself and is categorised as “Pure” (see next section) whereas item C6 presents a problem within a “real” context and is categorised as “Applied”. Although this latter item is not explicitly presented as a modelling task its successful completion will require simplifying assumptions to be made, such as dividing the profile into a number of separate sections, using mathematics to analyse each sub-section, before making some attempt to assess the validity of the final solution.

In summary, therefore, our instrument meets Bandura’s requirements, paying attention to the generality of the task, the level of its demand and strength of belief.

PILOT STUDY

The thirty self-efficacy items (ten at each level) were organised into three different versions of the questionnaire with each having link items ensuring that an item response theory (IRT[2]) model could potentially provide a good measurement model. The questionnaires were administered to a pilot sample of 341 students towards the

[2] The mathematical models of IRT calculate the probability of a correct response to an item as a function of the subject’s ability, the item’s difficulty and some other characteristics (depending on the relevant model)

end of their different AS courses in 23 different Further Education institutions across England.

Rasch analysis, and particularly the Rating Scale Model, was initially used to establish the validity of the instrument (Bond & Fox, 2001). The Rasch model, in its simpler form (i.e. the dichotomous model in which the responses are restricted to 1 and 0, or correct/incorrect) assigns an ability parameter to each student based on the number of his/her correct answers and a single difficulty parameter to each item, resulting from the number of students who answered that item correctly. Hence, it allows these estimates to be ordered in a common scale, using 'logit' as the measurement unit.

For this analysis we used the Rating Scale Model, which is the most appropriate for response categories in Likert type instruments that include ordered ratings such as ours ('not confident at all', 'not very confident', 'fairly confident', and 'very confident'). The Rating Scale Model (like any Rasch Model) also provides some fit statistics to indicate the fit of the data to the assumptions of the model, and particularly the dimensionality of the construct. Tests of fit aided the evaluation of the scalability of the item set and showed acceptable fit suggesting that our instrument could be used to describe the desired construct, i.e. perceived mathematical self-efficacy (Wright & Masters, 1982; Bond & Fox, 2001).

A next step in was to examine whether the items have significantly different meanings for the two groups, in which case differential item functioning (DIF) is present (as in technical guidelines suggested by Bond & Fox, 2001, p. 170-171).

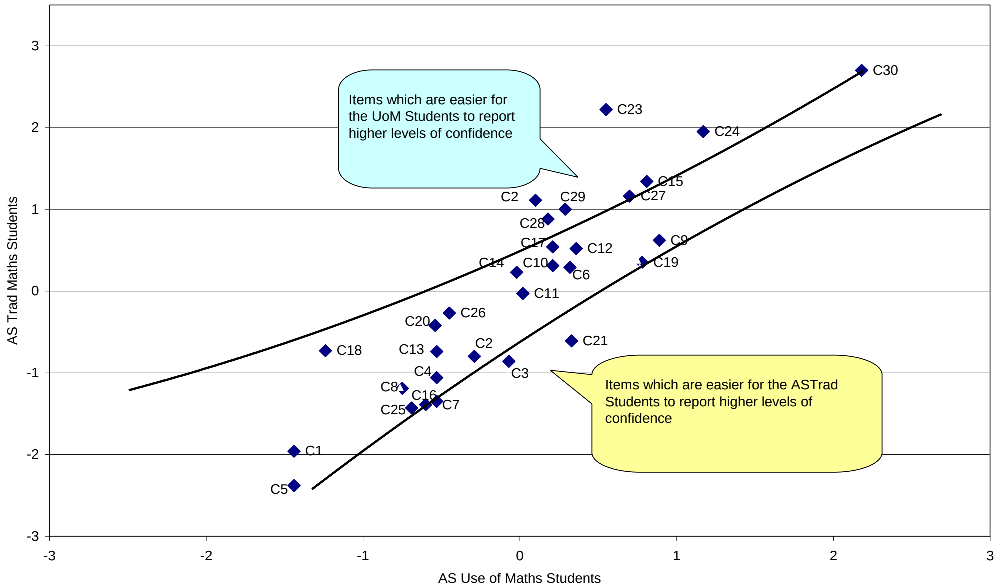


Figure 2. Item estimates (i.e. logits) for the two student groups (with the 95% confidence intervals)

The graph in Figure 2 plotted the difficulty of the items (in logits) as estimated for the two student groups separately, and the lines show the 95% confidence interval. According to the model's assumptions (and hence the 'ideal world's' scenario) it should be expected that these estimates should be invariant across the two groups, and all the items to fall inside the confidence intervals. In this case, however, it is demonstrated that certain items are outside these limits. Particularly, the items indicated at the top in Figure 2 are those that AS UoM students report to be significantly more confident in tackling than AS Trad students whilst AS Trad students report being significantly more confident than AS UoM students in tackling those at the bottom.

Given these findings, we hypothesized that there might be a latent sub-structure of the instrument that may be better modelled by a multi-dimensional model involving pure and applied dimensions. Focussing our analysis on those students with the lowest prior attainment (our target group of 70 AS UoM students and 93 AS Trad students) we performed multidimensional analysis for two different cases: (i) within-item multidimensionality (two dimensions), and (ii) between-item multidimensionality (two and three dimensions).

An instrument may, according to Wu, Adams and Wilson (1998), be considered multidimensional "*within – item*" if any of the items relates to more than one latent dimension. Within-item multidimensionality was determined by scoring the items on a 'Pure' and an 'Applied' demand on the basis of judgement about the nature of each item (i.e. items with no significant pure maths or applied maths demand, as judged by experts, were allocated to 'A' or 'P' respectively while those with some of each were allocated to both dimensions). From a different perspective *"between item multi-dimensionality*" occurs when an instrument contains several uni-dimensional sub-scales. In this case items were categorised into either two (2D model) or three (3D model) discrete categories as in Figure 3. Significantly these categorisations point to items C23 and C24, for example, as having an applied nature and C3 and C21 as having a pure nature, and the analysis points to the AS UoM students being more confident in tackling the former and the AS Trad students the latter.

Description	**3D model**	**2D Model**
no "real" context, may be solved using straightforward techniques	Pure (P)	Pure (P)
no "real" context (context not important to problem), requires decisions about approach to be taken		
problem in "real" context - method clear	Applied (A)	Applied / modelling (A/M)
problem in "real" context requires synthesis of a range of mathematical understanding / techniques		
requires assumptions / decision making in approach to solution	Modelling (M)	

Figure 3. Sub-scale categories used to investigate "between item multi-dimensionality" of the self-efficacy instrument.

Using a ConQuest (Wu, Adams and Wilson, 1998) multi-dimensional analysis (after Briggs and Wilson, 2003) we evaluated the comparison of the unidimensional and multi-dimensional models via a chi-square test on the deviance statistic (Figure 4). As they state *"because the multidimensional approach is hierarchically related to the unidimensional approach, the model fit can be compared relative to the change in the deviance value, where the difference in deviance between the two models is approximately distributed as a chi-square"* (p. 95) with the difference in the number of parameters as degrees of freedom, in each case.

MODEL	Deviance	Number of parameters
Unidimensional	5670.886	33
2D –within-item	5662.499	37
2D- between - item	5723.387	35
3D- between - item	5721.527	41

Figure 4. Comparison of the Unidimensional and Multi-dimensional models

Our analysis suggests that the within-item multidimensional model (as highlighted above) has a slightly better fit to the data than the uni-dimensional model, which in turn performs slightly better than either of the between-item models. This suggests that our instrument might be successfully used to not only measure perceived self-efficacy in maths overall, but should be able to identify sub-dimensions of perceived self efficacy in pure and applied maths.

FUTURE DIRECTIONS

Our initial study has allowed us to construct and validate an instrument that we can use to measure perceived self-efficacy in mathematics, as well as in constituent dimensions of "pure" and "applied" mathematics. This will be used to track changes longitudinally in the perceived self-efficacy of students "at the margins" following AS UoM and AS Trad courses in Colleges across England. Our focus, therefore, will be students with low prior attainment who in some institutions are even excluded from study of mathematics altogether. Their perceived self-efficacy, along with other outcome measures and student disposition to study in Higher Education in general as well as disposition to study mathematically demanding courses, will be surveyed at the start of their AS Mathematics courses (UoM and "Trad") as well as towards the end of the course and into the following year in a delayed post-test.

REFERENCES

Bandura, A. (1977). Self-efficacy: Toward a unifying theory of behavioural change. *Psychological Review*, *84*, 191 – 215.

Bandura, A. (1986). *Social foundations of thought and action: A social cognitive theory*. Englewood Cliffs, NJ: Prentice - Hall

Bandura, A. (1997). *Self efficacy: The exercise of control*. New York: Freeman.

Blum, W. (2002). ICMI Study 14: Applications and modelling in mathematics education – Discussion document. *Educational Studies in Mathematics, 51*(1/2), 149-172.

Bond, T. G., & Fox, C. M. (2001). *Applying the Rasch model: fundamental measurement in the human sciences*. Mahwah, NJ: Lawrence Erlbaum Associates.

Briggs, D. C., & Wilson, M. (2003). An introduction to Multi-dimensional Measurement using Rasch Models. *Journal of Applied Measurement, 4*(1), 87-100.

Hackett, G., & Betz, N.E. (1989). An exploration of the mathematics self-efficacy/ mathematics performance correspondence. *Journal for Research in Mathematics Education, 20*, 261-273.

Lent, R.W., & Hackett, G. (1987). Career self-efficacy: Empirical status and future directions. *Journal of Vocational Behaviour, 30*, 347-382.

Pajares, F., & Miller, M.D. (1994). The role of self-efficacy and self concept beliefs in mathematical problem solving: A path analysis. *Journal of Educational Psychology, 86*, 193-203.

Roberts, G. (2002). *SET for success: The supply of people with science, technology, engineering and mathematics skills.* London: HM Stationery Office.

Smith, A.: 2004, *Making Mathematics Count*. London: HM Stationery Office.

Wake, G.D. (2007) Considering workplace activity from a mathematical modelling perspective. In Blum W., Galbraith P.L., Henn, H.-W., & Niss M. (Eds.), *Modelling and Applications in Mathematics Education*. *The 14th ICMI Study.* (pp. 395 – 402). New York: Springer.

Williams, J. S., Wake G. D., & Jervis, A. (1999). General mathematical competence in vocational education. In C. Hoyles, C. Morgan & G. Woodhouse (Eds.), *Mathematics Education for the 21st Century* (pp. 90 – 103). London: Falmer Press.

Wright, B. D., & Masters, G. N. (1982). *Rating Scale Analysis*. Chicago: MESA Press.

Wu, M. L., Adams, R., & Wilson, M. (1998). *ACER ConQuest: Generalised Item Response Modelling Software*. Melbourne: Australian Council for Educational Research.