Pedagogy: the (mathematical) narrative in classroom practice

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Abstract

Central to individual students' emerging identities as learners and users of mathematics is their experience of mathematics in classrooms. Therefore in seeking to understand how we might engage more students in mathematics we focus on mathematics lessons, making observations and analyses in the ethnographic tradition with video and audio recordings. One focus of analysis investigates how teachers mediate the mathematics (i) using a range of different 'pedagogic practices', and (ii) by developing a lesson narrative interweaving mathematical with other, everyday narrative devices.

Immediately visible and apparently dominant in setting a 'tone' for the classrooms of different teachers are their pedagogic practices e.g. the monological transmission of information; whole-class discussions; 'modelling' an answer to an exam question; playing a 'game' in pairs; group problem solving, etc. These practices appear to differentially support relational or instrumental understanding of students whilst also developing classroom communities that are more or less engaging for different students.

To bring the mathematics more sharply into focus we turn to the socio-cultural construct of narrative (Bruner, 1996). Thus we conceptualise the teacher as storyteller engaging learners with his/her personal construction of a mathematical narrative of a topic/concept. From our case studies we analyse how each teacher organises the total "story" of their lesson using the "social narrative thread" in an attempt to connect with their students 'everyday' world and knowledge, running alongside, and at times interconnecting with, the "mathematical" world and 'scientific knowledge' (in Vygotsky's sense of the everyday and scientific).

We explore how in, two lessons where teacher colleagues introduce "the same" mathematics", these two strands of narrative (mathematics and social) may be aligned to a lesser or greater extent. We see how these may support each other in a powerful way to (re-) define what it is to do mathematics, and how a CHAT analysis suggests that this

realigns the object of the classroom activity system so that the community acts together to support the relational understanding of mathematics.

Setting the scene

As part of our study that investigated students' participation in post-compulsory mathematics at advanced level in England we observed their experiences in a range of classrooms in five case study colleges. It was immediately apparent that each classroom had its own distinctive feel with each teacher developing a distinctive environment for the learning of mathematics. As we shall report later in this paper we conceptualise the teacher as a narrator who in their development of the narrative of the lesson, and sequences of lessons weaves a complex story with a texture that includes both social and mathematical dimensions. Here we consider in particular the mathematical aspects of such narratives, and start our own narrative of this by describing two lessons we observed in one of our case study colleges by two teacher colleagues who were teaching two different classes the same topic. Whilst both of these teachers used similar pedagogic practices, which we describe as "connectionist", and consequently were next to each other on the scale of pedagogic practice that we developed within the project, it was apparent to the research team that there was something different about the "feel" of their different lessons. We reserve analysis and comment, and indeed discussion of our conceptualisation of narrative in lessons, until after our telling of the stories of Sally's and Tania's lessons.

Both lessons were early in the academic year, consequently bridging from previous learning to the start of the advanced course, focussing on quadratic functions. In each classroom the students sit in groups around clusters of tables. The walls are covered in posters that are their productions and those of other classes taught by their teacher: these are working documents rather than polished artefacts and are often used as aides-memoirs during lessons. A common practice during lessons is for each student to work on a mini whiteboard when asked by the teacher, often showing their working to the teacher when required, or sharing this within their group or with the whole class. The teaching, and therefore learner experience, contrasts starkly with that observed in many lessons at this level which consisted of almost entirely of "transmission" of information by the teacher.

Tania's lesson

With mini-whiteboards at the ready, Tania starts by asking students to draw a set of axes. She suggests using the squared side of the boards, and asks them to draw *any* quadratic graph. The mini – whiteboards allow Tania to quickly scan the room noting students' progress. After a short period she suggests, "if you're not sure what that word means wait a few seconds or you might see someone else on your table which might jog your memory….." She comments that everybody has drawn a "u-shaped" graph and enquires, "is that the only type of graph?" and goes on to ask the class to draw a different type of quadratic. Scanning the room she suggests that "we have either an egg shape or a u shape". Tania moves things quickly on telling the class that she has a quadratic drawn on her mini - whiteboard and asking if "you were to draw exactly the same quadratic, what information would you want me to give you?" A student responds immediately that he would want the "the *y*-intercepts, the *x*-intercepts….and .. er… the gradient". Tania asks what this student means by the gradient, and to fill the ensuing silence another student suggests some points on the graph. Tania says that she will describe the graph using some basic words: "it's a u-shaped graph", but then involves a student sitting nearby asking him how he would describe where it crosses the axes. He suggests it crosses twice on the *x*-axis and once on the *y*- axis. After some prompting he suggests that the *x*- axis intercepts are, "one positive, one negative". By now students are sketching their versions of this graph and Tania asks what else it would have been useful to know, to which everyone agreed that the position of the *y*- intercept. Tania summarises by drawing some sketches on the large whiteboard at the front of the room and elicits from a student that it would also be useful to know "the lowest point" and in the case of an "egg-shaped" curve "the highest point". Briefly, Tania explores the associated vocabulary with a student suggesting the word "peak" which she acknowledges as "a good word", before telling the class, "we do in fact call it the maximum". A student quickly points out that for the u-shaped graph this would be called the minimum and Tania suggests that this student knew exactly what she was going to ask next.

Tania now moves on to suggest that the lesson today is going to be about how to find these crucial points, "we are going to look at how we can get all that information ….and the crucial thing is going to be the way we write our quadratic equation". The activity that she uses to engage the students involves a worksheet (Figure 1) which has nine quadratic equations together with sketches on which the students will mark the details of the important points when Tania shows them drawn accurately using graph-plotting software on the interactive whiteboard. Tania then suggests that the students will work out how they can get this information from the way she has written the quadratic formula on the sheet, “pick out the coordinates from my graphs and then see if you can work them out from what is written down”.

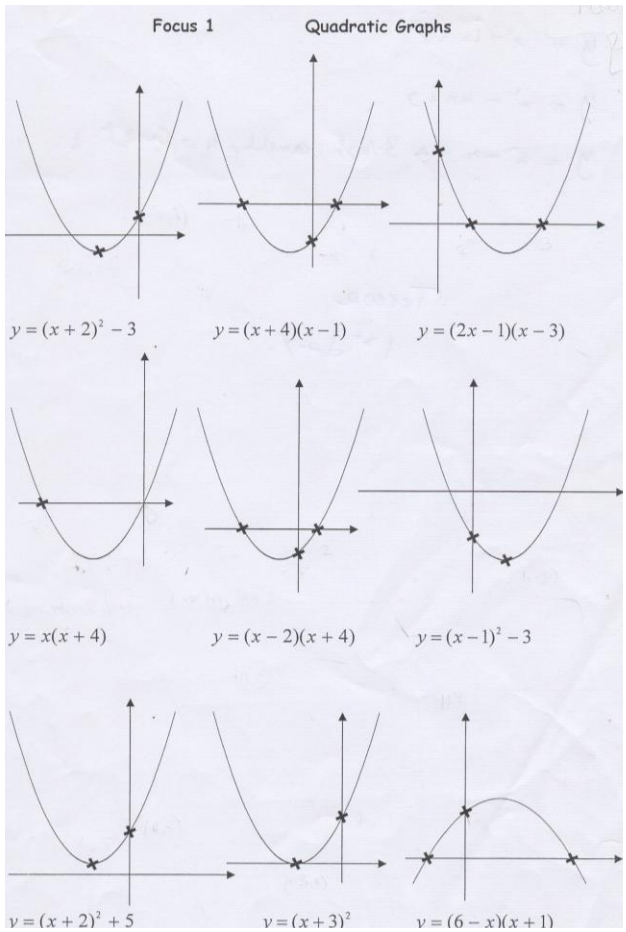


Figure 1: Worksheet used by students in Tania’s lesson

Tania leads the group carefully through the first few examples in the top row of the worksheet, ensuring that all of the students write down the *y*-intercept in each case and the *x*- intercepts or turning point as indicated on each sketch. Just before Tania is about to reveal the detailed fourth graph to the students she suggests that they might predict this one as the software is rather slow at bringing up the accurate plot. After completing the first six examples using the graph=plotting software, Tania suggests that she would like the group to predict information that they can tell from the equations of the final three using any patterns they notice from their work so far and she encourages discussion between pairs and groups about this.

After some 10 minutes Tania brings the whole group together returning to look at the graph of the first function, $y = (x + 2)^2 + 5$ (Figure 2) asking the students to circle each of the quadratics that are of this form on the work sheet. Referring to the function at the whiteboard, Tania, draws attention to the fact that there are two points marked that they had to find (see sketch at top left of worksheet in Figure 1), the minimum point and the *y*-intercept. She collects answers from a student who suggests the coordinates (- 2, 5) for the minimum point, who when asked to justify his answer, gives a rule that involves changing the sign of the value in the bracket for the *x*- value and taking the *y* – value directly from the remaining figure. A different student explained how to ascertain the intercept of the *y*- axis, with the student in this case explaining that one needs to substitute $x = 0$ into the expression to arrive at $y = 9$, and with Tania emphasising that this has come up many times during the course so far in the referring to work that the group had done on the straight line and circle.

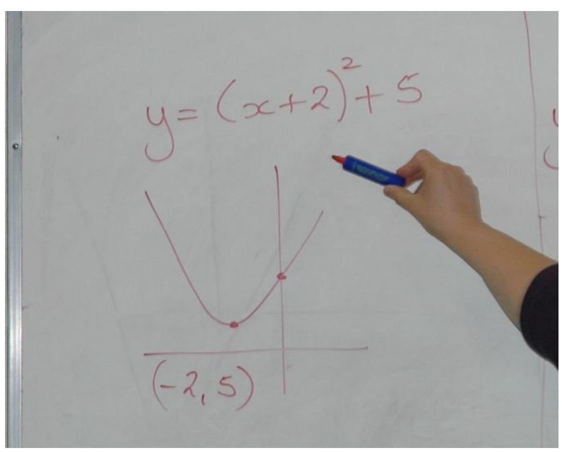


Figure 2. Tania working at the whiteboard

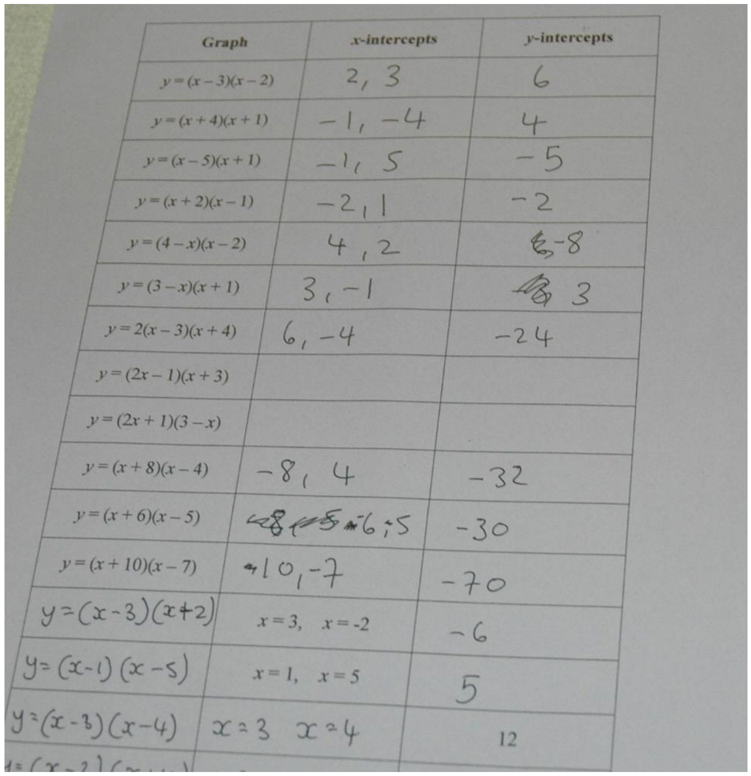
Sally's lesson

Sally starts her lesson in a very similar way to that of Tania: after ensuring that all students have a mini-whiteboard she asks them to draw a set of axes and shows them on the interactive whiteboard at the front of the room a graph of the basic function $y = x^2$. She then asks for a volunteer to come to the front of the room and describe to the rest of the class the drawing of a function similar to this, so that they can sketch its position on their axes. The volunteer comes to the front and suggests that the picture he has been given by Sally is of "*y* equals *x* squared" and it crosses the *y* axis at -2 point minus and the *x* axis at 0 and -1. Sally then asks the students to draw what they think the "picture" is. The students do this on the non-squared side of the whiteboards, each ending up with a rough sketch similar to the volunteer at the front, to whom they show their sketches by holding up their boards.

Sally asks for another volunteer to describe the position of another similar shaped graph, but introduces the restriction that no numbers can be used in the description. The student complies: making some gestures to indicate the axes he informs the class that, "on the x axis it crosses both times on the minus side" and "crosses the y axis up" presumably suggesting that there is a positive intercept on the *y*-axis. This continues for two more examples with further restrictions of not being able to use the words "positive" and "negative" which results in more gestures being used and the final example involving an "upside down" curve.

There follows some teacher led discussion about how knowing the "crossing points" on the *x*- and *y*- axes allows the students to position the curve: Sally prompts the students to remember the correct vocabulary (intercepts) for these significant points. At this point Sally introduces the "aims" of the lesson to the students showing them a pre-prepared screen on the interactive whiteboard.

She quickly moves into the next activity which will involve each student completing a worksheet (Figure 3) as she reveals, using graph plotting software, an accurate graph of each function on the interactive whiteboard. After making sure that everyone understands what they are to do by checking the responses to the first function ($y = (x - 3)(x - 2)$) Sally quickly moved through the next three functions and then asked the group to look at what was happening, to look for patterns in those done so far and then predict the intercepts for the next few prior to having a look at their graphs. After a few minutes she stops the class and asks what intercepts they had written down for the function $y = (4 – x)(x – 2)$.



Graph	*x*-intercepts	*y*-intercepts
$y = (x - 3)(x - 2)$	2, 3	6
$y = (x + 4)(x + 1)$	−1, −4	4
$y = (x - 5)(x + 1)$	−1, 5	−5
$y = (x + 2)(x - 1)$	−2, 1	−2
$y = (4 - x)(x - 2)$	4, 2	−8
$y = (3 - x)(x + 1)$	3, −1	3
$y = 2(x - 3)(x + 4)$	6, −4	−24
$y = (2x - 1)(x + 3)$		
$y = (2x + 1)(3 - x)$		
$y = (x + 8)(x - 4)$	−8, 4	−32
$y = (x + 6)(x - 5)$	−6, 5	−30
$y = (x + 10)(x - 7)$	10, −7	−70
$y = (x-3)(x+2)$	$x = 3$, $x = -2$	−6
$y = (x-1)(x-5)$	$x = 1$, $x = 5$	5
$y = (x-3)(x-4)$	$x = 3$ $x = 4$	12

Figure 3:Worksheet completed by students in Sally's lesson

A student suggests, "4 and 2" for the intercepts on the *x*-axis and "-8" on the *y*- axis. Before confirming these as being correct by showing a graph of the function, Sally, asks someone to explain their "rule". A student suggests that he has been finding the *x*-intercepts by changing the sign of each of the values in the brackets, and finding the *y*-intercept by multiplying these two values together. Sally now reveals the graph of the function confirming that the suggested intercepts (4 and 2) are correct. This she suggests provides some problems as the "rules" would give the *x*-intercepts as "–4" and "2" (which would lead to the correct *y*-intercept), or alternatively working with the suggested values, which were confirmed as correct by the graph plotting software, the "rule" for the *y*-intercept would give "8". Clearly there is a problem here with one or both of the rules.

At first Sally pursues this purely in terms of getting the "rule" to work. A student suggests that the example they are working on is different to the earlier examples as one of the brackets is $(4 – x)$ when it should be of the form $(x – a)$. He suggests that this can be achieved by rewriting as $-1 \times (-4 + x)$. This results in the complete function being $y = -1 \times (x-4)(x-2)$. Sally now explores with the class some of the reasoning behind the "rules" they have been applying asking how they would find the *x*- intercept if the function was that of a straight line. A student suggests putting *y* equal to zero, and Sally suggests therefore that in the case of the quadratic, in the form $y = (4-x)(x-2)$, one of the two brackets must be equal to zero: she laboured this point by asking a number of them to tell her two numbers that when multiplied together are equal to zero. She then asks what will be the value of *x* if the first of the two brackets in the expression is equal to zero, and having obtained the reply, "4", continues to elicit that for the second of the two brackets the value of *x* needs to be 2. Sally clarifies that this is what needs to be done to find where the quadratic crosses the *x*- axis.

Sally then returns to the formulation of the quadratic she thought had been suggested by one of the students, that is $y = -1 \times (x-4)(x-2)$, and suggests that this is an alternative way of looking at the function, "an upside down version" of the graphs that they had explored earlier.

The discussion then moves on to consideration of how to find the *y*- intercept: Sally asks how to find the *y* – intercept of any graph, and the immediate response from one student, taking the question to be specific to the case of the factorised quadratic, is to multiply the two *x*- intercept values together. Sally points out that this didn't work for the case that they have just been considering ($y = (4-x)(x-2)$) so another student suggests that they need to consider when *x* is zero. Lead by Sally, the group confirm that in this case the *y*-intercept will be at $y = 8$.

Lessons as narrative events

The descriptions of the lessons above focus on the mathematical development of the same topic in two different lessons and reflects the analytical frame we have used to attempt to make some sense of the complex interactions that take place in lessons. As you will have observed, Tania and Sally tell two subtly different stories about the mathematics at issue. We will come back to explore this further after considering the framework we have used in an attempt to understand lessons as narrative events.

In contrast to studies that seek to identify common normative “scripts” for mathematics lessons (eg, Wierzbicka, 1999), for example in large international studies such as TIMSS and PISA, we sought to identify *different* practices in both pedagogy and the structuring of mathematics. Our own observations, and in particular a measure of self-reported teacher practices that we developed (Pampaka et al, 2008), suggest that much teaching at this level might be described as conforming to a dominant “transmissionist” script; however, our case study colleges were chosen to allow us to explore deviation from such norms, as evidenced in the lessons of Tania and Sally.

In attempting to make sense of our observations in classrooms we have found useful the lens of “narrative”, in the sense of Ricouer, and as developed in educational settings by Bruner (1996) and others. We conceptualise the teacher as “narrator” revealing a mathematical plot whilst drawing on a range of pedagogic practices in an attempt to engage her audience in different ways. Pedagogic practices that we see being used range, for example, from monological transmission of information by the teacher with students taking notes through to activities where students work in groups, for example matching different representations of the same mathematical object on cards, with the teacher monitoring progress but seeming to take a back-seat role. These activities set an immediately obvious “tone” for the lesson, but as we have indicated here we have also been concerned to make some sense of the way in which the teacher, in addition to employing such practices, also organises the unfolding of *their* story of the mathematics. This is very much dependent on the individual teacher as we observe here with Tania and Sally who as colleagues, discuss their approach to a topic, but end up presenting different mathematical narratives which reflect their own understanding of the mathematical content as well as their beliefs about the nature of mathematics more widely. These we suggest reflect a teacher’s knowledge, understanding and beliefs about how mathematical knowledge might be structured and presented to students; such knowledge was termed pedagogic content knowledge (PCK) by Shulman in his seminal paper of 1986. Indeed, Bernstein (1996) and Pietig (1997) suggest that texts and mathematics respectively must increasingly be recognised as pedagogic. We build on this and in an attempt to understand how teachers structure mathematics for their students we consider the teacher as storyteller weaving together episodes in the development of a mathematical argument with each episode contributing to a significant plot. Further, we propose that with the lesson, or sequence of lessons, as our unit of analysis we can distinguish two strands of narrative: (i) a mathematical strand which is distinct and different from (ii) a 'social' strand that may incorporate details of context and attempts by the teacher to humanise the mathematics and engage students with it by their use of discourse and choice of pedagogic practices. This leads us to consider a two-dimensional framework (Figure 4): with one dimension taking account of the teacher's unfolding of the mathematics itself; and the other which is socially focused taking account of the pedagogic practices that they employ and the social discourse of the classroom (always heavily influenced by the teacher). The students' experience therefore reflects the intersection of these two dimensions: even if two teachers were to have the same mathematical narrative it is unlikely that their social narrative and choice of pedagogic practices would be the same and therefore students in their two classes would have different experiences even before the crucial issue of each individual student's interpretation of these experiences is taken into account.

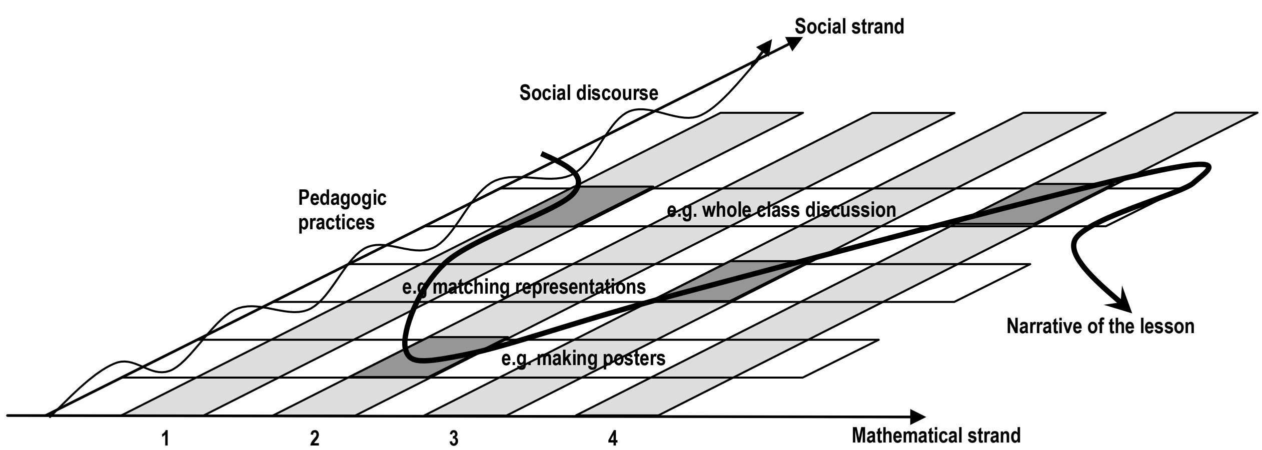


Figure 3: Schema illustrating two dimensional framework used to analyse the narrative of mathematics lessons

Whilst the social discourse relating to application and use of mathematics is clearly important in relation to understanding the students' engagement, we have explored the idea of there being different genre of mathematical narratives and we speculate that these may impact differently on different groups of students.

The mathematical narratives of the lessons of Tania and Sally

We turn now to draw some comparisons between the mathematical strands of the narratives of a Tania and Sally's lessons and in doing so contrast some of the key features of narrative in general, as identified by Bruner, (1996). In particular we examine how the narrator, in this case teacher, unfolds the crucial mathematical events in a temporal sequence, with a beginning (a problem to be solved), middle and end (resolution of this). of their choosing and note how they choose to introduce the same and different "mathematical characters" at different times. Note, for example, how Tania chooses to emphasise the coordinates of the maximum of minimum of the quadratic in contrast to Sally who chooses to this point to ignore this particular idea. However, of particular note is the way in which Sally introduces a problematic turn of events at an early stage: the algebraic formulation of the quadratic function with a maximum point $y = (4 - x)(x - 2)$. This, as the lesson description above shows, causes the students to focus in some depth to consider their "proceduralisation" of how to find the x and y intercepts of a quadratic function. Contrast this with Tania's lesson in which, as is evidenced by the worksheet and lesson description, the mathematical story is carefully structured so as to avoid "trouble" and so that it is easy to read by the students.

Bruner suggests that to be worth telling, narratives in general should run counter to expectancy and have "trouble" as a central feature. In mathematical narrative, particularly when the narrator wants to engage the audience in a dialogic pedagogy such a feature, a 'problematic' is essential, provoking different points of view from a shared understanding of an initial situation (Ryan & Williams, 2007). The very fact that students may have different points of view when exploring a problematic, will result, at least for some, in deviation from the expected. As Sally acknowledged in a series of interviews she often plans to introduce a major problematic that she sees as essential part of the mathematical narrative at an early stage of the lesson. In an interview after this particular lesson she discussed the crucial ingredients of her narrative of this topic:

> "….…. the y intercept I mean to some extent it's a red herring the y intercept and I could have left it completely out and just look at the x intercept. What I was getting out there was almost and maybe in retrospect

> they were too many I don't know… but in the planning it was to get away from the fact learning rules isn't going to work. This was another example of learning rules doesn't work so let's you know, think about what you know and lets be a mathematician, lets think about the y intercept because look learning rules doesn't work. Now, if I would do that lesson again with that class because of the huge amount of misconceptions I'd probably would leave the y intercept off. But if I still did that to a new class I'd put it back in, because the more I can get through to them the fact that: look we are not here to learn rules, we are here to think about the maths and how to do it and what it means and what it really is and what's really happening, then I think that is important because we're gonna have so many other things soon where if they are not careful they're gonna try the rules and they're going to get in another mess. So it was a reinforcement of that rather than why intercept itself is important."

Here, as in other lessons we observed and discussed with Sally, crucial in her narrative of, and about, mathematics is her belief about what it is to do mathematics and become a mathematician. Here she draws attention to the dangers of developing rules and procedures without asking, and understanding, why these work. When encouraging her students to explore mathematics in this way she talks of them as mathematicians and consequently adopting a particular approach that encourages connection making and deep rather than surface level. At a meta-level she develops an explicit narrative in relation to "doing mathematics" so much so that in lessons we observe students suggesting what to do next and this being closely aligned with Sally's planned activity (as her resources show). The social narrative of the classroom is also carefully controlled not only in terms of the pedagogic practices that Sally employs (remembering that we also see these in Tania's lesson) but also in the way the class as a community have been encouraged to behave. Discussing this in interview Sally says,

> "So, we kind of had that stage so then the focus is on respect and on working together as a team. And also there's the fact that it's not competition like who is going to pass or who is going to fail, we are working as a team, as a class, we're supporting each other and in doing that we've got to respect each other. That

actually takes quite a long time. Particularly that class today they've been more difficult than the other class, cause when they are all together if you didn't see them all together… but there's a lot of lads who have got a lot to say which actually often comes up, a lot of insecurities about their maths and think… if I am loud and say other things I can hide it. They are hiding behind that."

Crucial, therefore, also are the social interactions between the teacher and her students: we detect that whilst at times these may be spontaneous they are also often planned to interweave and interplay with the developing mathematical narrative in such a way that this social narrative may add power to and strengthen this.

In the extract of one of Sally's lessons that we describe here, we detect, therefore, not only the crucial introduction, at an early stage of the case of the quadratic expressed in the problematic form, but it is clear that Sally is developing social practices that will in the long term support the class in their becoming mathematicians: these social practices go beyond the pedagogic practices that both she and Tania use to engage students, as they develop rules of engagement in mathematical discourse that are aligned with a mathematical narrative which will problematise the mathematics being learnt at an early stage and where "struggle" is expected and valued. Whilst we detect similar pedagogic practices in Tania's classroom, which at first gives the classroom a similar feel, there is not the same strong alignment with the "problematic" in the mathematical narrative, the development of "rules" relating to engagement and discourse. We suspect that this is why, from our initial observations, we felt there was some significant difference in the teaching of the two colleagues.

In terms of CHAT our analysis suggests that in most classrooms, particularly in post-16 settings individual students are focused on meeting the objective of success in assessment leading to "qualification". In most classrooms the division of labour is clear with the teacher taking the central role, setting many of the rules, cultural, expectations and norms. However, equally, many of these are culturally and historically bound: at a surface level this is evident from inspection of curriculum specification and assessment practices that are little different from those of fifty years or more ago (perhaps the annual debate about "standards" ensures that there is likely to be little deviation from the expected "norm" from year to year!). At a deeper level, however, classroom practices are perhaps even more stable, with much teaching and learning being little different now than it was many years ago. However, in these two classrooms we see two teachers each working with new instruments in terms of pedagogic practices, in ways that might prove more engaging and motivating for students. It is in the classroom of Sally, however, where there has been a crucial realignment of the object of activity: as Sally herself points out she develops a community which works to support each other in deep learning and understanding of mathematics. Certainly at this point of the year she downplays the importance of assessment, certification and qualification that as we report elsewhere are dominant in the narrative of many teachers and indeed the institutions in which they work. Sally achieves this by paying attention to both social and mathematical strands of her narrative, as well as developing new rules of engagement to support this new learning community. As we also point out Sally is perhaps unusually (hopefully not uniquely) positioned to become the "expert" teacher that she undoubtedly is: we hope that others might also be able to adopt expert practices informed by a narrative and CHAT conceptualisation of their classroom activity.